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Bibliometry and Data Analytics
Research Article

Unveiling patterns and trends in research on cumulative damage models for statistical and reliability analyses: Bibliometric and thematic explorations with data analytics

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Abstract

This study comprehensively explores the research landscape within statistical and reliability studies, focusing on the Birnbaum-Saunders distribution, Gaussian inverse distribution, cumulative damage models, and fatigue life prediction. Using a combination of bibliometric analysis, network visualization, thematic mapping, and latent Dirichlet allocation, we analyze 465 articles from the ISI Web of Science database. These articles were selected for their relevance based on a targeted search strategy. Our analysis identifies key trends, collaboration networks, and emerging research themes. Notable growth in scholarly activity was observed from 2015 to 2021, with a peak around 2021, followed by a decline in the number of publications. Relevant contributions were noted from countries such as Brazil, Canada, Chile, China, Iran, Japan, and the United States. The thematic analysis of keywords reveals influential motor themes like the Birnbaum-Saunders distribution and expectation-maximization algorithm; specialized niche areas such as producer risk; emerging or declining themes like the generalized Birnbaum-Saunders distribution; and foundational themes including cumulative damage and fatigue life distributions. A cluster analysis states key focus areas, such as material durability and advanced statistical methods. Integrating latent Dirichlet allocation, six main topics are derived, capturing broad thematic structures. However, some niche areas do not align directly due to their specialized nature and limited cross-field impact. These findings map the current research on this thematic and suggest future research directions, including deeper exploration of niche themes, integration of advanced statistical methods in practical applications, and increased collaboration across diverse research areas to enhance the robustness and applicability of reliability models.

Keywords: Bibliometrical analysis· Birnbaum-Saunders distribution· cumulative damage· fatigue life prediction· Gaussian inverse distribution· latent Dirichlet allocation.

Mathematics Subject Classification: Primary 62E15 · Secondary 94A16.

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1. Introduction

Statistical analysis and reliability studies have witnessed important advancements over the years, with particular interest in areas such as the Birnbaum-Saunders distribution (Birnbaum and Saunders, 1968, 1969a,b), Gaussian inverse distribution, cumulative damage models, and fatigue life prediction. However, there remains a gap in the comprehensive synthesis of these research areas, especially concerning their interconnectedness and emerging trends. This gap highlights the need for a systematic exploration to map out the current research landscape, identify predominant themes, and uncover future research directions.

A substantial body of literature has focused on various aspects of the Birnbaum-Saunders distribution and its applications in reliability and lifetime analysis. Notable works include studies on the statistical properties and inference for the Birnbaum-Saunders distribution (Ng et al., 2003; Park and Padgett, 2005a; Balakrishnan et al., 2007; Kumar el al, 2008; Cordeiro and Lemonte, 2011; Mazucheli et al., 2018; Chaves et al., 2019; Bourguignon and Gallardo, 2022), as well as its application in reliability (Masatoshi, 2002; Murakami and Miller, 2005; Gómez et al., 2009; Guiraud et al., 2009; Balakrishnan et al., 2011; Fatemi and Shamsaei, 2011; Paula et al., 2011; Azevedo et al., 2012; Marchant et al., 2013; Leiva et al., 2014; Marchant et al., 2019; Sánchez et al., 2021). These studies have contributed to understanding the applicability of the Birnbaum-Saunders distribution in modeling lifetime data and predicting material reliability (Bhattacharyya and Fries, 1982; Durham and Padgett, 1997; Yao and Himmel, 2000; Ng et al., 2003; Park and Padgett, 2005a; Balakrishnan et al., 2007; Kumar el al, 2008; Cordeiro and Lemonte, 2011). Additionally, innovative approaches and applications of the Birnbaum-Saunders distribution have been explored in recent works, including semi-parametric additive modeling (Cárcamo et al., 2024), construction of bivariate models with Birnbaum-Saunders marginals (Arnold et al., 2021, 2023), frailty models (Gallardo et al., 2024), Bayesian computation for fatigue data (Leiva et al., 2022), and applications in insurance claim size distribution and other topics (Caro-Lopera et al., 2012; Leiva et al., 2014; Gómez-Déniz et al., 2022).

Another critical area of investigation involves the Gaussian inverse distribution, particularly its application in reliability analysis and statistical modeling. Seminal papers have explored the theoretical foundations and practical implementations of the Gaussian inverse distribution (Bhattacharyya and Fries, 1982; Shang and Yao, 1999; Yao and Himmel, 2000; Shi and Mahadevan, 2001; Sanhueza et al., 2008), highlighting its efficacy in various engineering applications (Leiva et al., 2007b; Kundu et al., 2008; Balakrishnan and Kundu, 2018). These investigations have underscored the distribution versatility and importance in reliability studies (Cheng and Plumtree, 1998; Leiva et al., 2007a).

Cumulative damage models and fatigue life prediction represent another major focus within the reliability domain. Research in this area has primarily concentrated on developing and refining models to predict material fatigue and cumulative damage under various loading conditions (Fatemi and Yang, 1998; Aid et al., 2011; Risitano et al., 2013; Lv et al., 2014; Rege and Pavlou, 2017). Pioneering studies have also employed machine learning methods to enhance fatigue life prediction accuracy (Bhatti, 2010; Zhu et al., 2019), demonstrating the potential of advanced computational methods in reliability engineering (Park and Padgett, 2005b; Lio et al., 2009; Costa et al., 2021).

In addition to these core themes, the literature has also addressed statistical methodologies and their applications in reliability. Key publications have discussed inference and probabilistic models, which are crucial for reliability analysis (Rieck and Nedelman, 1991; Cheng and Plumtree, 1998; Leiva et al., 2007a; Lemonte et al., 2007; Kundu et al., 2008; Lio et al., 2009; He et al., 2019). These works have provided a foundation for developing statistical methods tailored to reliability and lifetime analysis (Leiva et al., 2008; Kundu et al., 2010; Mazucheli et al., 2021).

Other contributions include optimal sample size determination (Costa et al., 2021), quantile regression for bounded data (Mazucheli et al., 2021), and environmental applications (Puentes et al., 2021; Reyes et al., 2021).

Recent technological and methodological advancements have driven substantial progress in these fields. For instance, the development of sophisticated statistical models and computational methods has enabled more accurate predictions of material fatigue and failure, enhancing the reliability and safety of engineering systems (Murakami and Miller, 2005; Gómez et al., 2009). Despite these advancements, a comprehensive synthesis that bridges the gap on these research areas remains scarce, particularly concerning their interconnectedness and emerging trends (Masatoshi, 2002; Lv et al., 2014). This gap highlights the need for a systematic exploration to map out the current research landscape, identify predominant themes, and uncover future research directions (Baklizi et al., 2004; Lemonte et al., 2007). To address this gap, the present study has the following objectives: (i) To characterize the current research landscape in statistical and reliability studies, providing a comprehensive overview of its state-of-the-art; (ii) To identify predominant themes and collaboration patterns, revealing the main areas of focus and the nature of research partnerships within these domains; and (iii) To uncover future research directions and untapped opportunities by spotlighting emerging trends and potential innovations in statistical and reliability studies.

Employing a combination of bibliometric analysis, network visualization, thematic mapping, and latent Dirichlet allocation (LDA), this research analyzes a carefully curated selection of 465 scientific articles. Initially, we conducted a comprehensive search using the ISI Web of Science database to capture relevant literature. The search terms were chosen to encompass key areas of interest: Birnbaum-Saunders distribution, Gaussian inverse distribution, cumulative damage models, and fatigue life prediction. Specifically, we used search criteria that combined these terms to ensure a broad yet focused retrieval of articles. From the resulting dataset, we selected the 500 most relevant articles based on relevance ranking provided by the database. After removing duplicates, 465 articles were retained for in-depth analysis. This comprehensive approach ensures a robust examination of the current research landscape, highlighting key trends and relevant contributions within the field.

After this introduction, the present article is structured as follows. Section 2 describes the methodology adopted for our analysis. In Section 3, we present our detailed analytical framework. Section 4 delves into the insights obtained from the data analytics. Section 5 concludes with a discussion of the implications and future directions for research.

2. Methodology

Mathematical and statistical methods are paramount in ensuring the reliability and validity of scientific research. In this study, we employ these methods to conduct bibliometric, network, and thematic analyses on the collected literature. This section delineates the mathematical and statistical underpinnings of our analysis, enabling a comprehensive examination of the interrelations between the research topics specified in our query.

2.1. Bibliometric analysis

Bibliometric analysis is an essential tool in academic research, facilitating the investigation of patterns, relationships, and trends within scholarly articles. This analysis leverages quantitative indicators to outline the evolution and focal areas of research domains. Furthermore, it quantifies and maps research trends within the field, identifies key authors, institutions, and countries, and reveals collaboration networks and the most influential articles. Through these indicators, bibliometric analysis provides a holistic overview of the academic landscape, offering insights into the dynamics of research activity and intellectual production.

In bibliometric analysis, the term *corpus* refers to the collective body of text or documents under examination. This corpus forms the dataset from which patterns of term usage, thematic prevalence, and academic linkages are derived. The corpus can vary in size and scope, ranging from a targeted collection of articles within a specific discipline to extensive databases covering multiple fields of study. These collections and databases of documents form the basis for extracting insights into scholarly discourse and evolving trends within the research community, in our case, related to the topics specified in our query.

Network analysis provides a framework for dissecting the interactions and configurations within our corpus (Newman, 2010). This analysis allows us to uncover networks of collaboration, thematic interconnections, and structural patterns that underpin academic inquiry within the specified research topics.

In the context of bibliometric and network analyses, *terms* refer to relevant words or phrases that encapsulate key concepts or themes within the dataset. These terms are extracted from the titles, abstracts, and keywords of scholarly articles, serving as critical elements in mapping the thematic landscape of the corpus.

Within network analysis, *nodes* are representations encapsulating the thematic elements central to each study. Nodes can represent entities such as authors, keywords, or articles, depending on the focus of the study. Each node represents an element of the corpus, allowing for the exploration of connections and relationships within the academic discourse.

The nodes are interconnected by *edges*, which denote relationships such as co-authorship, citation, or thematic similarity. Edges indicating co-occurrences or relational ties facilitate the visualization of the internal structure and thematic landscape of the literature. This visual mapping aids in identifying key themes, influential entities, and potential areas for future investigation, providing an understanding of the dynamics at play.

2.2. Co-occurrence matrix

The co-occurrence matrix is a critical tool in network analysis, quantifying the strength and frequency of the relationship between pairs of terms within the corpus. It is constructed by tabulating the number of times each pair of terms appears together in the same context, such as within a single document or abstract. This matrix serves as the foundation for constructing the network graph, where the edges between nodes are weighted based on the co-occurrence frequencies. Analyzing the co-occurrence matrix enables researchers to discern patterns of thematic linkage and conceptual association, further illuminating the intricate web of connections that constitute the scholarly discourse on the specified themes.

2.3. Graph representation

This representation provides a mathematical structure to model relationships within the corpus. A graph is defined by $G = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$ represents the set of vertices (or nodes) and $\mathcal{E}$ the set of edges that connect these vertices, symbolizing the relationships between them, such as thematic correlations or co-occurrences. Weights may be assigned to edges to reflect the frequency or strength of these relationships, enhancing the granularity of the network analysis and enabling a more detailed exploration of the thematic landscape within the domain of study.

2.4. Node degree

In network analysis, the node degree serves as an indicator of connectivity within the network, representing the number of connections (edges) a node has. For a node v, its degree, denoted as $\text{degree}(v)$, can be calculated by counting the number of edges incident to v.

Formally, if we let $G = (\mathcal{V}, \mathcal{E})$ be a graph with $\mathcal{V}$ as the set of vertices and $\mathcal{E}$ as the set of edges, the degree of v is given by

$$\text{degree}(v) = |\{e \in \mathcal{E}\colon v \in e\}|,$$

where $|\cdot|$ denotes the cardinality of a set, meaning the total number of edges incident to v. This indicator is crucial for understanding the importance and centrality of nodes within the network. A node with a high degree is indicative of a key concept or entity with strong interconnectivity to other nodes, showing its central role in the network.

2.5. Path length and diameter

Understanding the interconnectedness within the literature on our theme benefits from analyzing the path length and network diameter. The path length signifies the shortest series of connections (edges) linking two nodes, which can represent concepts or entities within the field. Mathematically, the metric of path length between two nodes u and v in a graph G, denoted as $\text{dist}(u, v)$, is defined as the minimum number of edges required to connect them. This is often assumed under the condition that each edge has a unit weight for simplification. The network diameter is the longest of all the shortest path lengths within the network, formulated as

$$\text{diam}(G) = \max_{u,v \in \mathcal{V}} \text{dist}(u, v),$$

where $\mathcal{V}$ is the set of all nodes in the network. This metric, indicative of the network overall spread, provides insights into the broad scope of research thematic. A smaller diameter suggests a tightly knit research area, whereas a larger diameter indicates a more diverse and extensive body of work.

2.6. Network analysis

In network analysis, identifying denser groups of nodes, clusters or communities, within the network is essential for analyzing its structure. The Louvain (Blondel et al., 2008) and Girvan-Newman (Girvan and Newman, 2002) methods are widely used for this purpose. These methods segment the network into clusters based on the density of connections between nodes, facilitating the study of the network inherent groupings and relationships.

The Louvain method optimizes the modularity of a network, quantifying the division of a network into communities. Modularity is calculated employing the density of edges within clusters, compared to the expected density of edges if connections are distributed at random, given the network structure. Modularity is defined as

$$Q = \frac{1}{2m} \sum_{ij} \left(A_{ij} - \frac{k_i k_j}{2m} \right) \delta(c_i, c_j),$$

where A_{ij} represents the edge weight between nodes i and j; k_i and k_j are the sum of the weights of the edges attached to nodes i and j, respectively; m is the sum of all of the edge weights in the network; c_i and c_j are the communities of nodes i and j; whereas δ is the Kronecker delta function, which equals 1 if $c_i = c_j$ and 0 otherwise.

Initially, the Louvain method assigns each node to its own cluster. Then, it iteratively relocates nodes to different communities, if doing so results in an increase in the network overall modularity. This local optimization is performed for all nodes until no further increase in modularity is achievable by moving nodes.

After this local optimization phase, the Louvain method aggregates nodes within the same cluster and constructs a new reduced network. In this network, nodes represent the previously identified communities. The method reapplies the optimization process to this reduced network, continuing iteratively until no further increase in modularity is possible. This iterative process efficiently identifies hierarchical organization within large datasets.

Using the Louvain method, our study uncovers thematic communities within the research network related to the specified query. Each cluster, defined by its internal connections, represents a distinct thematic area, showing the breadth from foundational concepts to specific applications. This method clarifies the field thematic structure and highlights areas for further research (Fortunato, 2010).

2.7. Quantitative measures of network and community structure

To evaluate the internal coherence and influence of each cluster within the network, we compute density and centrality indices. The density of a cluster reflects its internal connectivity and is calculated as

$$\mathrm{dens}_{\mathrm{com}} = 100 \times \frac{\sum_{x\in \mathrm{com}} \sum_{y\in \mathrm{com}, y\neq x} O_{xy}}{T_{\mathrm{com}}}, \tag{2.1}$$

where x and y are terms within the cluster, "com" say, O_{xy} represents the number of co-occurrences between terms x and y, and T_{com} is the total number of term pairs in the cluster. High values of $\mathrm{dens}_{\mathrm{com}}$ indicate a closely related thematic group.

The centrality of a cluster (Freeman, 1978) measures its relative importance within the overall network and it is quantified as

$$\mathrm{cen}_{\mathrm{com}} = 10 \times \sum_{x\in \mathrm{com}} \sum_{y\notin \mathrm{com}} O_{xy}, \tag{2.2}$$

where O_{xy} is the number of co-occurrences between term x within the cluster com and term y in different clusters. Higher values of $\mathrm{cen}_{\mathrm{com}}$ suggest a theme of greater relevance within the broader research landscape.

2.8. Thematic mapping

Thematic mapping, supported by co-word network analysis and clustering methods, is an approach that enables the systematic visualization and analysis of the corpus thematic structure. This approach reveals predominant themes, emerging areas of interest, and topics that have received less attention. A thematic map, generated from a co-occurrence matrix, visualizes the conceptual structure and evolution of research topics based on term co-occurrences.

In our study, thematic maps act as strategic diagrams, positioning themes within four quadrants based on their centrality and density metrics, as defined in Equations (2.2) and (2.1). The application of these metrics within the framework of thematic maps draws from the methodology proposed by Callon et al. (1991) to interpret their relevance and development within the research field. In this context, the use of these metrics aids in categorizing themes into four distinct quadrants – motor themes, niche themes, peripheral themes, and basic themes – each representing different stages of development and influence within the field of cumulative damage research. The x-axis represents centrality, reflecting a theme interaction level with other themes, whereas the y-axis is density, indicating the theme internal cohesion. The interpretation of quadrants on thematic maps is as follows:

- Motor themes (upper-right quadrant) —High centrality and density, indicating well-developed and impactful themes.
- Niche themes (upper-left quadrant) —High density but lower centrality, showing specialized themes with strong internal but weaker external connections.
- Peripheral themes (lower-left quadrant) —Low in both centrality and density, representing emerging or declining themes.
- Basic themes (lower-right quadrant) —Relevant themes with lower density, indicating broad and foundational topics.

This thematic structure, identified through the application of quantitative measures like those detailed in Equations (2.1) and (2.2) to our thematic maps, facilitates an understanding of the research landscape concerning the topics in our study.

2.9. Latent Dirichlet allocation for topic modeling

LDA is a sophisticated method used to identify, extract, and organize thematic content from large volumes of textual data (Blei et al., 2003). By employing a probabilistic model, LDA creates a structured framework for systematically analyzing literature on specific topics (Tang et al., 2014). LDA views documents as mixtures of various topics, with each topic characterized by a specific distribution of words. This representation allows documents to be modeled as combinations of multiple topics in varying proportions. The generative process for any document d within a corpus D, illustrated in the flowchart of Figure 1, has the following steps:

Step 1. Determine the number of words N in document d by choosing $N \sim \text{Poisson}(\lambda)$ to model the variability in document lengths across the corpus.

Step 2. Establish the distribution of topics within document d by selecting $\theta_d \sim \text{Dirichlet}(\alpha)$, where θ_d specifies the proportional presence of each topic within the document.

Step 3. For each word position n in the document, where $n \in \{1, \ldots, N\}$:

3.1 Assign a topic z_n to the word at position n by choosing $z_n \sim \text{Multinomial}(\theta_d)$, reflecting the document thematic composition.

3.2 Generate word w_n for position n based on the selected topic z_n, by picking w_n from the probability $p(w_n|z_n, \beta)$, conditional on the chosen topic z_n and governed by β.

Step 3.2 completes the document generation by specifying which word from the topic word pool fills each position in the document. Steps 1-3 provide an understanding of how LDA conceptualizes documents as compositions of topics, underpinning the model utility in extracting and organizing thematic content from extensive textual data.

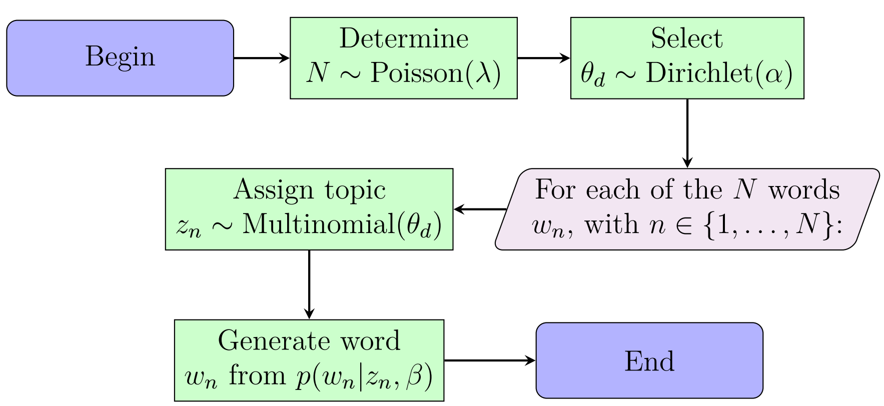

Figure 1. Flowchart of the generative process of LDA.

Essential components of the LDA model include the following elements:

- λ —Parameter of the Poisson distribution modeling document length variability.
- α, β —Parameters of the Dirichlet prior distributions for the per-document topic distributions and the per-topic word distributions, respectively.
- θ_d —The topic distribution within a particular document d.
- z_n —The topic assigned to word n in document d.
- w_n —The specific word chosen based on topic z_n and the distribution of β.

The parameter α influences the document-topic distribution within the LDA framework such that:

- A small value of α results in documents with a sparser topic distribution, dominated by fewer topics.
- A large α value promotes a more even distribution of topics within documents, enhancing thematic diversity.

In our analysis, we employ a symmetric Dirichlet prior for α, setting it to a constant value of $1/K$ for all topics, where K denotes the total number of topics. This choice facilitates a balanced initialization, allowing the model to adjust dynamically and learn the complexity of topic distributions directly from the data. The influence of the Dirichlet distribution on the topic distributions within documents is captured by

$$p(\theta_d|\alpha) = \frac{\Gamma(\sum_{i=1}^{K} \alpha_i)}{\prod_{i=1}^{K} \Gamma(\alpha_i)} \theta_{d1}^{\alpha_1 - 1} \times \cdots \times \theta_{dK}^{\alpha_K - 1},$$

where θ_d represents the vector of topic probabilities for document d. Similarly, the parameter β shapes the topic-word distribution within the LDA framework such that:

- A small β value encourages the model to construct topics from a narrower set of words, increasing topic distinctiveness.
- A large β value fosters a more uniform distribution of words across topics, enhancing thematic overlaps.

We use a symmetric Dirichlet prior distribution for β, assigning it a uniform value of $1/V$, where V represents the size of the vocabulary. This usage establishes an equal initial probability for each word in the vocabulary to be generated by any topic, with refinements occurring as the model learns from the textual data. The role of the Dirichlet distribution in shaping the word distributions for each topic k is mathematically expressed as

$$p(\phi_k|\beta) = \frac{\Gamma(\sum_{j=1}^{V} \beta_j)}{\prod_{j=1}^{V} \Gamma(\beta_j)} \phi_{k1}^{\beta_1 - 1} \times \cdots \times \phi_{kV}^{\beta_V - 1},$$

where ϕ_k represents the probability distribution over words for topic k.

With the foundational parameters α and β established, we transition to the inference phase of the LDA model. This phase is critical for uncovering the latent thematic structure within our corpus. We utilize Gibbs sampling, a Markov chain Monte Carlo algorithm, which is effective for estimating topic and word distributions. This sampling simplifies the inference process by focusing on sampling the topic assignments z_n for each word w_n within each document d. Collapsed Gibbs sampling streamlines the inference by concentrating on the direct assignment of topics to individual words, reducing model complexity and accelerating convergence.

The Gibbs sampling iteratively collects from the conditional distribution of topic assignments, given all other current topic assignments as

$$p(z_i = k|\boldsymbol{z}_{-i}, \boldsymbol{w}) \propto \left(\frac{n_{-i,k}^{(w_i)} + \beta}{n_{-i,k}^{(m)} + V\beta} \right) \left(\frac{n_{-i,k}^{(d_i)} + \alpha}{n_{-i}^{(d_i)} + K\alpha} \right), \tag{2.3}$$

where $\propto$ indicates proportionality rather than exact equality, z_i the topic assignment for word i in the dataset, and $\boldsymbol{z}_{-i}$ the set of all topic assignments excluding word i. The term $\boldsymbol{w} = (w_1, \ldots, w_m)$ denotes the collection of words in the documents. The number of co-occurrences of word w_i assigned to topic k, excluding the current assignment i, is given by $n_{-i,k}^{(w_i)}$, while $n_{-i,k}^{(m)}$ is the total number of words assigned to topic k, excluding the current word. Furthermore, $n_{-i,k}^{(d_i)}$ is the number of words in document d_i assigned to topic k, also excluding the current assignment. We recall that V represents the size of the vocabulary, K the number of topics, and α, β are the parameters of the Dirichlet prior distributions on document-topic and topic-word distributions, respectively.

The implementation of this sampling strategy is outlined in Algorithm 1, which iteratively applies the conditional distribution specified in Equation (2.3) to update topic assignments across the corpus, aiming to discover the most probable topic distribution that represents the underlying thematic structure of the documents. The flowchart of this process is depicted in Figure 2.

Algorithm 1: Collapsed Gibbs Sampling for LDA.

Input: Corpus of documents D, number of topics K, parameters α and β, number of iterations T
Output: Topic assignments for each word in the corpus
Initialize topic assignments z_i for each word randomly.
for $t = 1$ to T **do**
  for each document $d \in D$ **do**
    for each word w_i in document d **do**
      Remove current topic assignment z_i for word w_i.
      Update numbers $n_{-i,k}^{(w_i)}$, $n_{-i,k}^{(m)}$, and $n_{-i,k}^{(d_i)}$ excluding current assignment.
      Sample new topic z_i for word w_i based on the conditional probability given by Equation (2.3).
      Update numbers $n_{-i,k}^{(w_i)}$, $n_{-i,k}^{(m)}$, and $n_{-i,k}^{(d_i)}$ including new assignment.
    end for
  end for
end for
return Topic assignments z_i for all words

The LDA model effectiveness hinges on its capacity to categorize texts into a predetermined number of topics, denoted by K. Choosing the optimal K influences the detail and granularity of the topics discovered and also the coherence and interpretability of the model output. To determine the most suitable K, we rely on two main metrics:

- Coherence score (C_v) —It quantifies the semantic similarity between the most relevant words within each topic. A higher C_v value indicates greater semantic coherence, suggesting that the top words of the topic are more meaningful when interpreted together. The measure C_v combines the normalized pointwise mutual information and cosine similarity between word vectors by means of

$$C_v = \frac{1}{N} \sum_{i<j} \log \left(\frac{p(w_i, w_j) + \varepsilon}{p(w_i)p(w_j)} \right),$$

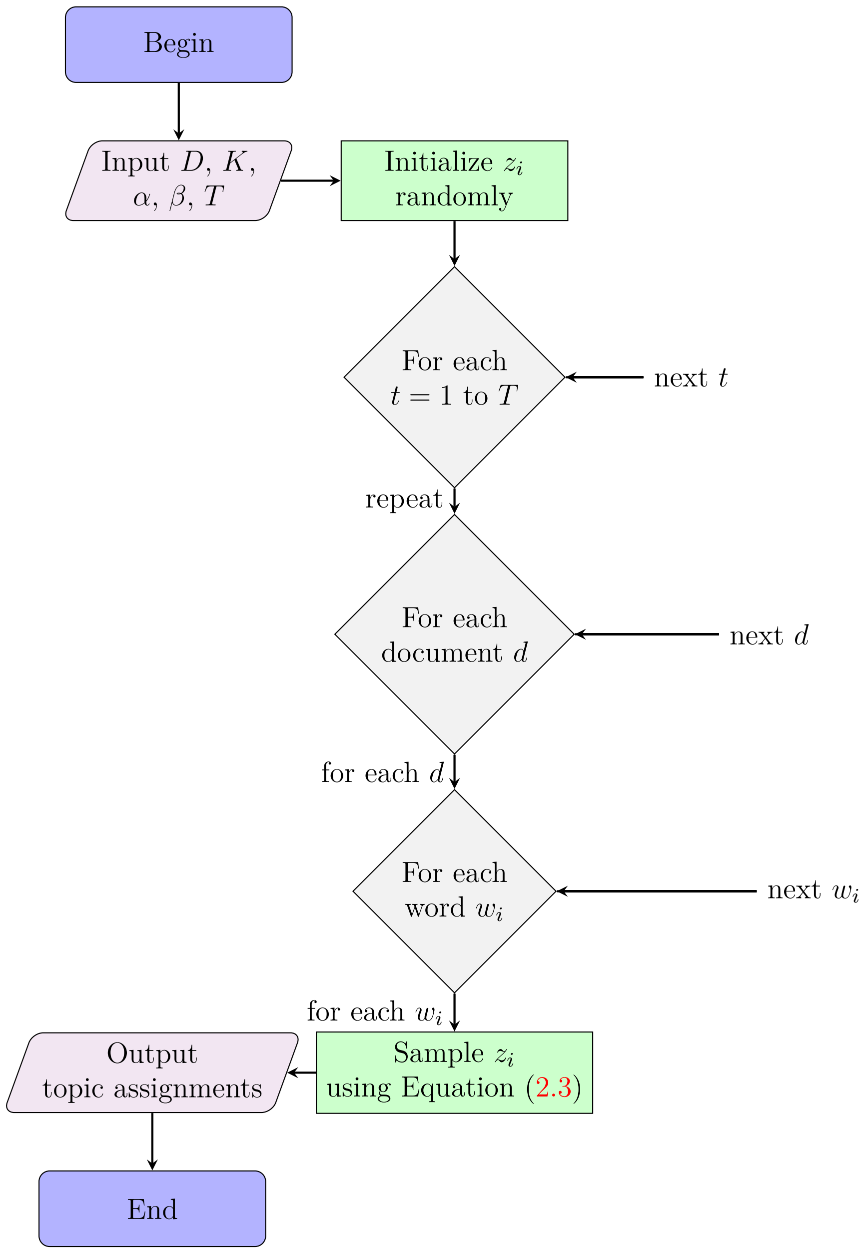

Figure 2. Flowchart outlining the steps of the collapsed Gibbs sampling method for LDA.

where $p(w_i, w_j)$ is the probability of words w_i and w_j occurring together, $p(w_i)$ and $p(w_j)$ are the probabilities of words w_i and w_j occurring independently, and ε is a small positive value to avoid division by zero.

- Model perplexity —It measures the predictive capability of the model on unobserved data, with lower values indicating a better fit. It is formally defined as the exponential of the negative normalized probability of a held-out test set given by

$$\text{perplexity}(D_{\text{test}}) = \exp\left(-\frac{\sum_{d=1}^{M} \log(p(\boldsymbol{w}_d))}{\sum_{d=1}^{M} N_d}\right),$$

where D_{test} is the held-out test set, $\boldsymbol{w}_d$ the words in document d, M the number of documents in the test set, and N_d the number of words in document d.

Concluding our methodological exposition as well as encompassing bibliometric, network, and thematic mapping, and topic analyses through LDA, we now pivot towards the practical application of these analyses in understanding the research landscape of our area of interest.

3. Analytical framework: Exploring themes on the topic

This section contains the practical exploration of the research landscape within the statistical and reliability domains, focusing on Birnbaum-Saunders distribution, Gaussian inverse distribution, cumulative damage models, and fatigue life prediction. We analyze the current research on the mentioned domains, identifying prevailing themes, collaboration networks, and emerging trends.

3.1. Systematic review

A systematic review and a bibliometric analysis were conducted to explore the specified topics using the query `(birnbaum AND saunders) OR (gaussian AND inverse) OR (cumulative AND damage) OR (fatigue AND life)` across the ISI Web of Science database. The ISI Web of Science was chosen for its comprehensive coverage of high-quality peer-reviewed journals across various scientific disciplines. Its robust indexing and citation tracking capabilities make it an invaluable resource for identifying influential research and emerging trends. Utilizing this database ensures that our analysis captures a wide breadth of relevant studies, providing a thorough overview of the current research on the topic.

Given the broad scope of our query, the initial search results were extensive. To ensure a focused and impactful analysis, we decided to limit our study to the 500 most relevant articles as determined by the relevance ranking provided by the database. This limitation was chosen to capture the most pertinent and influential works in the field, thereby ensuring that our analysis reflects the forefront of research and key contributions.

After removing 35 duplicates, we refined our corpus to 465 articles for in-depth analysis. This curated collection forms the foundation of our subsequent thematic and topic modeling analyses. By concentrating on relevance, we aimed to distill the literature to its most important elements, providing a robust basis for identifying key trends, influential publications, and major contributors within the specified domains.

3.2. Exploratory data analysis

The bibliometric analysis began with an examination of the trends in the volume of articles published over time. Using the 500 most relevant articles, the range of publications spans from January 1982 to May 2024. As illustrated in Figure 3, there has been an increase in scholarly activity over the years, indicating a growing interest and engagement with these subjects. The data reveal an important upward trend from the years 2015 to 2021, followed by a decline in the number of publications from 2021 onwards. This pattern suggests a peak in research activity around 2021, with subsequent fluctuations in interest or shifts in research focus.

To identify the primary outlets for research dissemination, we examined the journals with the highest number of publication. Understanding where the majority of research is published helps to highlight key venues that importantly contribute to the field development. Identifying these journals provides insights into the main channels through which new findings are disseminated and which journals are central to the academic conversation in this area. Table 1 lists the top 20 journals by publication number.

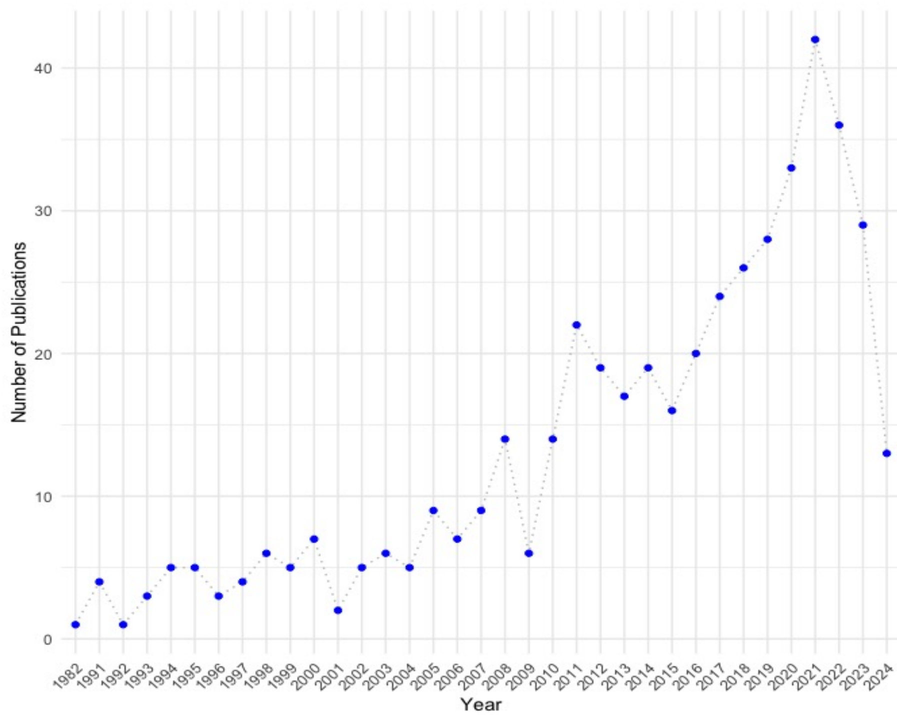

Figure 3. Distribution of articles per year.

Table 1. Top 20 journals distributed by the number of publications.

Journal	Publisher	Number
International Journal of Fatigue	Elsevier	42
Computational Statistics and Data Analysis	Elsevier	28
Fatigue and Fracture of Engineering Materials and Structures	Wiley	26
Journal of Statistical Computation and Simulation	Taylor and Francis	23
Communications in Statistics: Theory and Methods	Taylor and Francis	20
Communications in Statistics: Simulation and Computation	Taylor and Francis	17
Journal of Applied Statistics	Taylor and Francis	13
Symmetry	MDPI	12
Journal of Statistical Planning and Inference	Elsevier	11
IEEE Transactions on Reliability	IEEE	10
Statistics and Probability Letters	Elsevier	10
Applied Stochastic Models in Business and Industry	Wiley	9
Mathematics	MDPI	9
International Journal of Damage Mechanics	SAGE Publications	7
Applied Sciences	MDPI	6
Brazilian Journal of Probability and Statistics	Brazilian Statistical Association	6
Journal of Multivariate Analysis	Elsevier	6
Metrika	Springer	6
Quality and Reliability Engineering International	Wiley	6
Statistics	Taylor and Francis	6

To further understand the impact and dissemination of research, we organized the selected articles based on the number of citations. This approach highlights the most influential works and provides insights into which studies have had the greatest impact on the research community. Table 2 lists the top 20 articles distributed by the number of citations, showing seminal works that have shaped the field.

Analyzing Table 2 reveals that certain journals consistently publish highly impactful research. Notably, the *International Journal of Fatigue* stands out, with multiple entries among the top-cited articles, indicating its relevant role in the dissemination of research related to fatigue life prediction and cumulative damage models. This journal appears frequently, highlighting its prominence in the field and its influence on advancing knowledge in these areas.

Other key journals include *Computational Statistics and Data Analysis* and *Communications in Statistics: Simulation and Computation*, both of which are well-represented in the top-cited articles list. These journals are pivotal in publishing influential research on statistical methods, particularly those involving the Birnbaum-Saunders and Gaussian inverse distributions. Additionally, journals such as *Technometrics* and *Engineering Fracture Mechanics* also have notable entries in the top-cited articles list. These journals reflect their important roles in publishing high-impact research on statistical modeling and fracture mechanics, respectively. The presence of these journals underscores their contribution to the field and their role in shaping ongoing research discussions.

Table 2. Top 20 articles distributed by the number of citations.

Authors	Title	Journal	Year	Citations
Fatemi A, Yang L	Cumulative Fatigue Damage and Life Prediction Theories: A Survey of the State of the Art for Homogeneous Materials	International Journal of Fatigue	1998	940
Park C, Padgett WJ	Accelerated Degradation Models for Failure Based on Geometric Brownian Motion and Gamma Processes	Lifetime Data Analysis	2005	319
Fatemi A, Shamsaei N	Multiaxial Fatigue: An Overview and Some Approximation Models for Life Estimation	International Journal of Fatigue	2011	294
Balakrishnan N, Leiva V, López J	Acceptance Sampling Plans from Truncated Life Tests Based on the Generalized Birnbaum-Saunders Distribution	Communications in Statistics: Simulation and Computation	2007	203
Rieck JR, Nedelman JR	A Log Linear-Model for the Birnbaum-Saunders Distribution	Technometrics	1991	173
Murakami Y, Miller KJ	What is Fatigue Damage? A View Point from the Observation of Low Cycle Fatigue Process	International Journal of Fatigue	2005	149
Ng HKT, Kundu D, Balakrishnan	Modified Moment Estimation for the Two-Parameter Birnbaum-Saunders Distribution	Computational Statistics and Data Analysis	2003	145
Kuroda M	Extremely Low Cycle Fatigue Life Prediction Based on a New Cumulative Fatigue Damage Model	International Journal of Fatigue	2002	112
Shang DG, Yao WX	A Nonlinear Damage Cumulative Model for Uniaxial Fatigue	International Journal of Fatigue	1999	112
Kumar R, Gardoni P, Sánchez-Silva M	Effect of Cumulative Seismic Damage and Corrosion on the Life-Cycle Cost of Reinforced Concrete Bridges and Earthquake Engineering	Structural Dynamics	2009	111
Shi P, Mahadevan S	Damage Tolerance Approach for Probabilistic Pitting Corrosion Fatigue Life Prediction	Engineering Fracture Mechanics	2001	107
Kundu D, Kannan N, Balakrishnan N	On the Hazard Function of Birnbaum-Saunders Distribution and Associated Inference and Computational Statistics	Data Analysis	2008	106
Subramanian S, Reifsnider KL, Stinchcomb	A Cumulative Damage Model to Predict the Fatigue Life of Composite Laminates Including the Effect of a Fiber-Matrix Interphase	International Journal of Fatigue	1995	103
Cordeiro GM, Lemonte AJ	The β-Birnbaum-Saunders Distribution an Improved Distribution for Fatigue Life Modeling	Computational Statistics and Data Analysis	2011	102
Leiva V, Barros, Paula, Galea	Influence Diagnostics in Log-Birnbaum-Saunders Regression Models with Censored Data	Computational Statistics and Data Analysis	2007	100
Aid A et al.	Fatigue Life Prediction under Variable Loading Based on a New Damage Model and Materials	Design	2011	100
Yao WX, Himmel N	A New Cumulative Fatigue Damage Model for Fibre-Reinforced Plastics	Composite Science and Technology	2000	91
Cheng G, Plumtree A	A Fatigue Damage Accumulation Model Based on Continuum Damage Mechanics and Ductility Exhaustion	International Journal of Fatigue	1998	87
Lv, Huang, Zhu, Gao, Zuo	A Modified Nonlinear Fatigue Damage Accumulation Model	International Journal of Damage Mechanics	2015	87
Leiva V, Barros, Paula, Sanhueza	Generalized Birnbaum-Saunders Distributions Applied to Air Pollutant Concentration	Environmetrics	2008	87

The geographical distribution of the studies provides insight into global research efforts within our domain. As depicted in Figure 4, the analysis shows a broad range of countries, indicating widespread interest in these topics across different regions. Brazil leads with a total of 104 articles, followed closely by China with 103. The United States contributes 48 articles, Canada 36, and Iran 23. Other notable contributors include Chile with 18 articles and Japan with 17. This distribution highlights the relevant research output from these countries, reflecting their active role in exploring statistical and reliability topics.

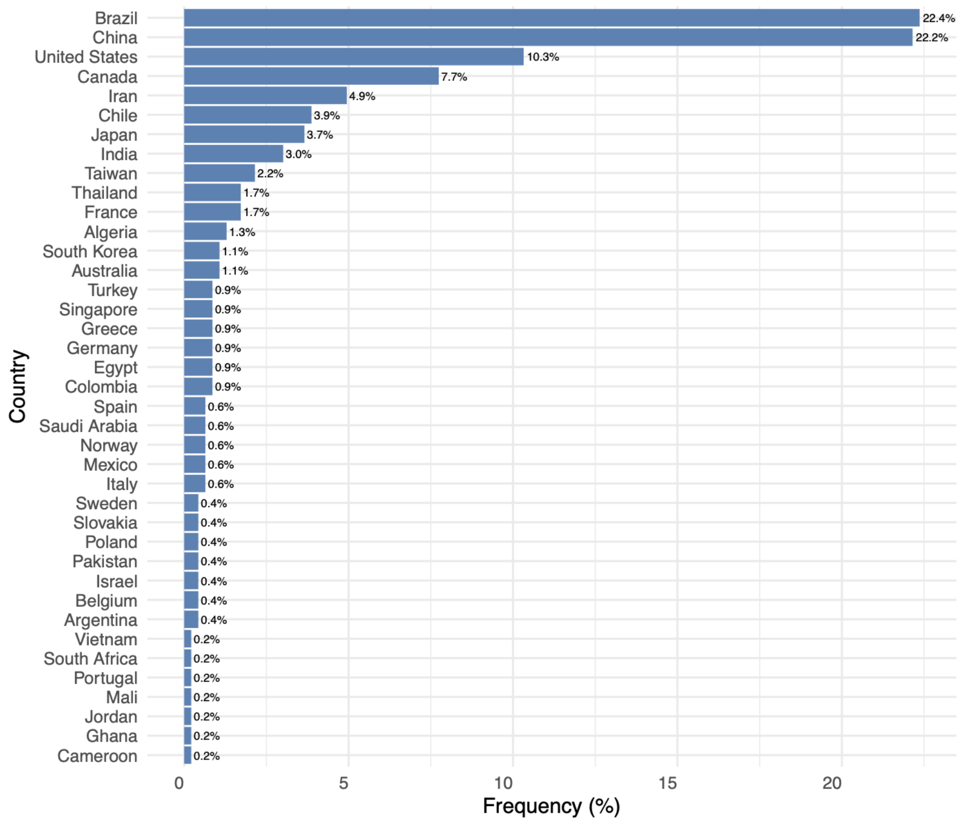

Figure 4. Distribution of articles by country.

The distribution of articles by continent further emphasizes the global interest in these research topics. As summarized in Table 3, Asia leads with 197 articles (42.37%), followed by South America with 128 articles (27.53%), and North America with 87 articles (18.71%). Europe contributes 34 articles (7.31%), Africa 14 articles (3.01%), and Oceania 5 articles (1.08%). This distribution shows the diverse geographic engagement and the importance of these research topics across different regions. Figure 5 shows the concentration of research efforts in Asia and South America, indicating strong regional focuses on these topics. The substantial contributions from North America and Europe also reveal the global relevance and collaboration in statistical and reliability studies based on cumulative damage models.

Table 3. Distribution of articles by continent.

Continent	Number	Percentage
Africa	14	3.01
Asia	197	42.37
Europe	34	7.31
North America	87	18.71
Oceania	5	1.08
South America	128	27.53

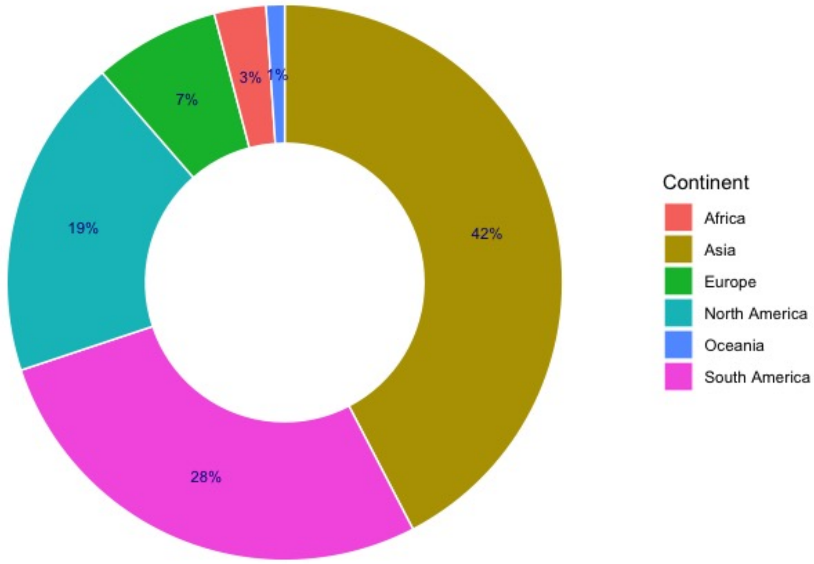

Figure 5. Distribution of articles by continent.

3.3. Network analysis

Following the geographic distribution analysis, we further explored the collaborative dynamics within the specified research areas. Understanding the collaborative network is crucial, as it reflects the interdisciplinary and international nature of research efforts in this area. To identify the collaborations, we extracted all countries mentioned in the affiliations of the authors and analyzed the co-authorships between different countries. This process involved identifying all countries listed in the author affiliations and determining all possible pairs of collaborating countries. We then counted the frequency of each country pair to identify the most frequent collaborations. As depicted in Figure 6, the collaboration network analysis highlights the most frequent collaborations between countries. The analysis reveals that the most frequent collaborations occur between countries with relevant research output in these areas. Notably, the highest frequency of collaboration is between Brazil and Chile with 37 co-authored articles, followed by Iran and Mali with 18, and Brazil with Canada and Colombia, both with 10 articles each.

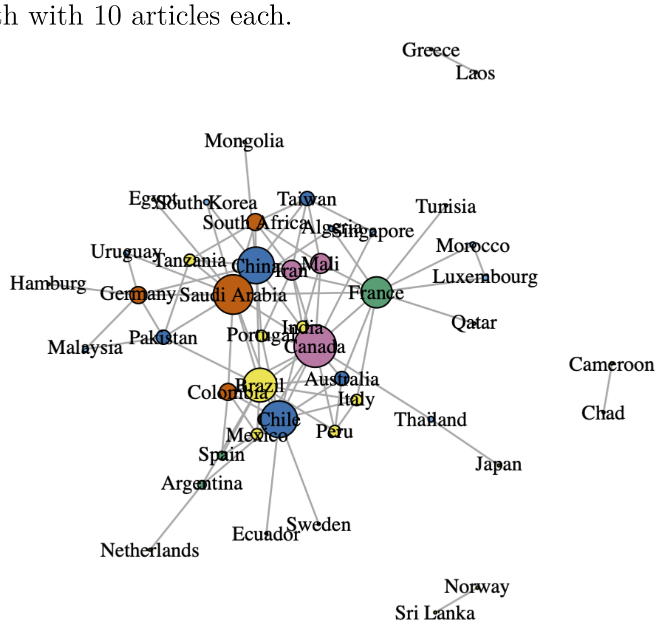

Figure 6. Collaboration network based on co-authorships between countries.

Table 4 summarizes the top collaborations, showing the frequency of co-authorship between country pairs. These findings highlight the active collaboration networks that exist between countries, facilitating the exchange of knowledge and driving forward the research agenda in statistical and reliability studies. These findings underscore the importance of international collaboration in advancing research in these fields. The presence of strong bilateral partnerships highlights how countries with relevant research outputs work together to push the boundaries of knowledge and innovation.

The network visualization reveals a complex web of collaborations spanning multiple countries, showing the global interest and shared efforts towards advancing research in statistical and reliability studies. Key nodes such as Brazil, Chile, Canada, China, France, and Saudi Arabia emerge as central hubs of collaboration, indicating their key role in fostering international research partnerships. These collaborations extend beyond high-output countries to include nations with fewer articles, suggesting an inclusive research environment that encourages knowledge exchange and mutual growth. The diversity of the network highlights the universality of these research topics and their potential to bring together researchers from varied backgrounds and disciplines.

This analysis of the collaborative landscape enriches our understanding of how the scientific community is converging around these topics. It provides valuable insights into the synergies that drive research progress and innovation. Such collaborations demonstrate the vibrant research community commitment to exploring the potential of these topics to revolutionize the domain.

Table 4. Top collaborations by frequency of co-authorship.

Country pair	Frequency
Brazil-Chile	37
Iran-Mali	18
Brazil-Canada	10
Brazil-Colombia	10
Canada-China	9
Chile-Colombia	8
Canada-Chile	7
Canada-India	7
Iran-South Africa	6
Canada-Saudi Arabia	5
Canada-Iran	4
Canada-Mali	4
China-Mali	4
Mali-South Africa	4
Singapore-Taiwan	4
Algeria-France	3
Brazil-Peru	3
Chile-Italy	3
Chile-Portugal	3
Chile-Spain	3

3.4. Thematic analysis of keywords

To deepen our understanding of the research area within the specified domains, we conducted a thematic analysis of keywords. This analysis identifies the most prevalent themes and concepts explored by researchers. Table 5 presents the identified themes, offering insights into the focal points of current studies. This table provides centrality and density indices for each theme, offering insights into their influence and coherence within the research network.

Table 5. Density and centrality indices of identified themes.

Theme	Frequency	Centrality	Density
cumulative damage	15	9.0	4.0
Birnbaum-Saunders distribution	20	11.0	7.0
estimation	4	5.0	2.0
simulation	2	6.0	9.5
cumulative fatigue damage	4	7.0	1.0
em algorithm	19	10.0	11.0
fatigue life	4	8.0	3.0
producer risk	1	1.5	9.5
generalized Birnbaum-Saunders	1	1.5	5.5
cumulative	2	3.0	8.0

The thematic map presented in Figure 7 visually represents the relationships between various themes. By highlighting both the development (as represented by density) and relevance (evidenced by centrality) of each theme, the map facilitates a comprehensive understanding of the dynamics and potential areas for innovation in this field of study. The thematic map is divided into four quadrants, each representing clusters of related research topics. The position of a theme within a quadrant provides insight into its role and interconnections within the broader research landscape, with the quadrants defined as:

- First quadrant (upper right - motor themes) —This quadrant includes terms such as *em algorithm* and *Birnbaum-Saunders distribution*. These themes are well-developed and highly influential within the field, indicating they are both extensively researched and central to the network. The high density and centrality of these themes underscore their pivotal role in advanced statistical methodologies and their application to lifetime data modeling and reliability analysis.
- Second quadrant (upper left - niche themes) —Featuring terms like *producer risk* and *cumulative*, this quadrant highlights specialized areas with strong internal connections but less influence across the broader field. These themes are crucial for specific aspects of reliability and risk assessment, representing focused research areas that are important but not as widely interconnected with other topics. Notably, *simulation* appears between the first and second quadrants, indicating its transitional nature and relevance to both motor and niche themes.
- Third quadrant (lower left - emerging or declining themes) —This quadrant includes *generalized Birnbaum-Saunders* and *estimation*. Themes in this quadrant are either gaining traction or losing relevance, as indicated by their lower centrality and density. The presence of these themes suggests ongoing research and development, with potential for increased integration into the broader field as their relevance evolves.
- Fourth quadrant (lower right - basic themes) —This quadrant encompasses terms such as *cumulative damage*, *fatigue life*, and *cumulative fatigue damage*. These themes are fundamental and essential for the field structure, characterized by high centrality but lower density. They provide the groundwork for more specialized research, highlighting broad interest areas that are foundational to understanding material fatigue and damage accumulation.

Combining quantitative keyword analysis with qualitative visual representation provides a robust understanding of research trends and focal points within the field. These combined insights are essential for guiding future research directions, fostering collaboration, and addressing the evolving challenges and opportunities in statistical and reliability studies.

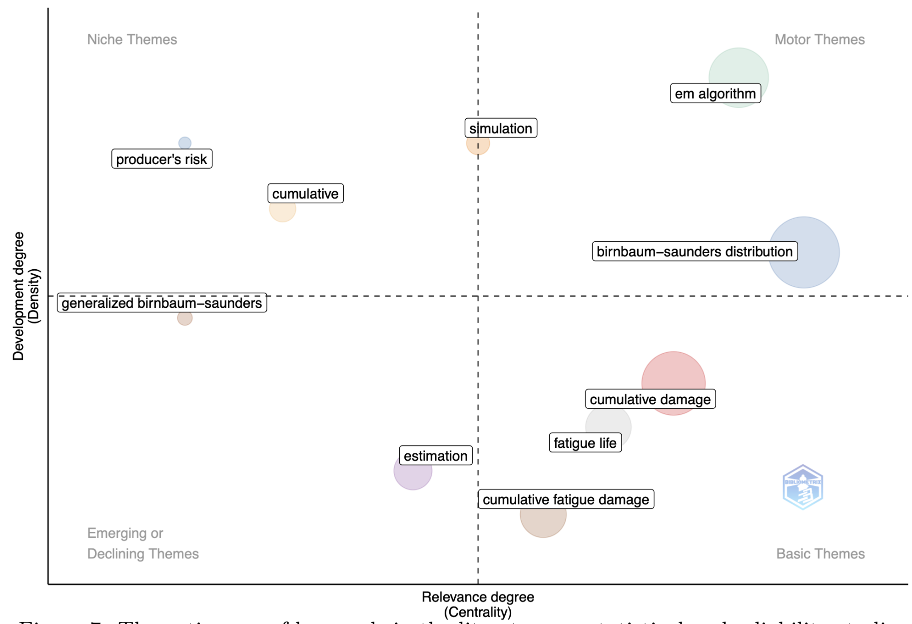

Figure 7. Thematic map of keywords in the literature on statistical and reliability studies.

3.5. Cluster analysis of keywords

To further understand the research landscape, we utilized the Louvain method to identify clusters within the keyword network. This method revealed 10 distinct clusters, but we focus on the major ones for clarity. The visualization of these clusters using the Fruchterman-Reingold layout is shown in Figure 8, where vertices are colored according to their cluster membership. The sizes of the major clusters, and the subsequent descriptions provide insights into their thematic focus, where Clusters 1, 2, 3, 4, and 5 have 129, 100, 73, 49, and 48 terms, respectively.

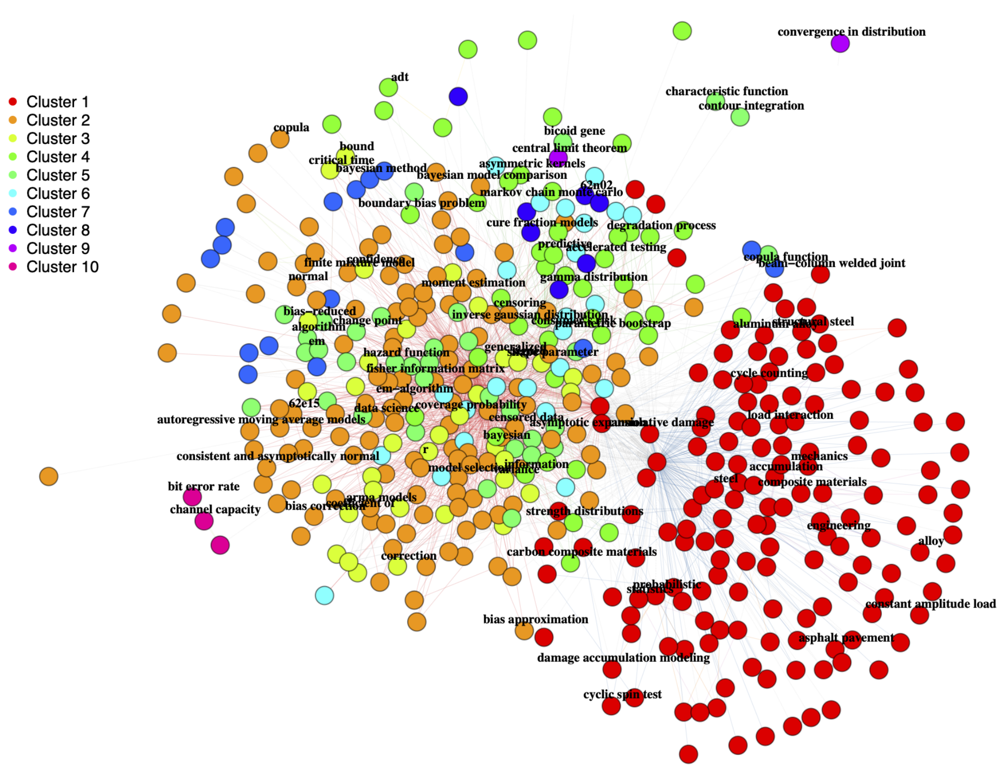

Figure 8. Visualization of the keyword network.

Detailed descriptions of the major clusters are the following:

- Cluster 1 —This cluster, the largest with 129 terms, includes keywords such as *atmosphere corrosion*, *computational modelling*, *continuum damage*, *corrosion*, *creep compliance*, *criteria*, *critical plane*, *cumulative laws*, *cumulative seismic damage*, *engineering*, *evolution*, *lifetime data*, *loading sequence effect*, *monte-carlo simulation*, and *welded joints*. It represents research focused on material durability, cumulative damage assessment, and reliability analysis. The high frequency and centrality of these terms indicate their foundational importance in engineering and materials science, emphasizing long-term performance and failure prediction.
- Cluster 2 —With 100 terms, this cluster centers on keywords such as *asymptotic expansion*, *bartlett-type correction*, *bias correction*, *birnbaum saunders distribution*, *bimodal distribution*, *simulations*, *case-deletion model*, *chain majorization*, *characteristics function*, *data dependent over time*, *ecm-algorithm*, *global and local influence*, *hazard rate*, *information matrix*, *multivariate*, *reparameterization*, and *test*. These terms highlight advanced statistical methods and algorithms crucial for lifetime data analysis and reliability. The methodological rigor in this cluster reflects its relevance in statistical modeling and computational statistics.
- Cluster 3 —This cluster, with 73 terms, features keywords like *akaike information criterion*, *bivariate*, *bootstrap bartlett correction*, *bound*, *correction*, *estimates*, *generalized*, *modified*, *outliers*, *pivotal quantity*, *point estimation*, and *probability coverage*. This indicates a strong focus on estimation methods and probabilistic models, particularly in contexts requiring robust statistical inference and simulation-based approaches. The prominence of terms like *confidence* and *variance* suggests extensive research into the accuracy and reliability of statistical estimates.
- Cluster 4 —Containing 49 terms, this cluster includes keywords such as *accelerated*, *accelerated degradation test*, *accelerated testing*, *asymmetric kernel*, *boundary bias problem*, *degradation process*, *failure*, *gamma distribution*, *inverse gaussian distribution*, *simulation*, *tensile strength*, and *bayes factor*. It represents research on specific statistical distributions used in reliability engineering and life testing. The inclusion of *damage models* and *accelerated testing* indicates a focus on modeling degradation processes and predicting material failure under accelerated conditions.
- Cluster 5 —With 48 terms, this cluster is characterized by keywords such as *asymptotic normality*, *bias-corrected modified moment estimators*, *characteristic function*, *consistent estimators*, *em*, *local influence*, and *log linear models*. It highlights research on statistical diagnostics, influence measures, and specialized statistical distributions. This cluster focus on terms like *hazard function* and *generalized leverage* suggests a deep dive into reliability metrics and robust statistical analysis.

The cluster analysis, combined with the thematic map of Figure 7, allow us to understand the research trends and focal points within the field. By examining the clusters, we can see that Cluster 1 emphasizes material durability and cumulative damage, which are critical for long-term performance and failure prediction in engineering applications. Cluster 2 highlights advanced statistical methods and algorithms, reflecting the importance of methodological rigor in lifetime data analysis and reliability. Cluster 3 focus on estimation methods and probabilistic models underscores the need for robust statistical inference. Cluster 4 deals with specific statistical distributions used in reliability engineering, indicating an interest in modeling degradation processes under accelerated conditions. Cluster 5 emphasis on statistical diagnostics and global/local influence measures suggests a detailed exploration of reliability metrics. These insights are crucial for guiding future research, fostering collaboration, and addressing evolving challenges in statistical and reliability studies, ensuring that efforts are aligned with the most impactful and emerging areas in the field.

4. Implementation and insights from topic modeling

In this section, we delve into the application of topic modeling to identify and analyze the latent thematic structures within our research dataset. This process aids in uncovering underlying patterns and trends that are not immediately evident from individual articles.

4.1. Methodology and model configuration

The exploration of latent topics was conducted through an analysis of 500 articles that, once filtered, reached 465 articles. We utilized their titles, abstracts, and keywords. This analysis aimed to map out the thematic landscape related to our research focus. Using the `topicmodels` package of the `R` software (Grün and Hornik, 2011), we applied LDA to our textual dataset. Each document analyzed by the LDA model represents a compilation of the article title, abstract, and keywords. This approach ensures a comprehensive thematic representation of each article. The text from the title, abstract, and keywords was concatenated into a single textual representation for each article. The analysis proceeded through the following stages:

- Data preprocessing —Prior to model fitting, the dataset underwent preprocessing to enhance textual uniformity and model performance by converting all text to lowercase to standardize the dataset, removing punctuation, numbers, and common English stopwords to reduce noise and focus on meaningful content, and combining titles, abstracts, and keywords into a single text per document to ensure a holistic representation of each article thematic content.
- Document-term matrix construction —A document-term matrix was constructed from the preprocessed text, capturing the frequency of terms across documents. This matrix served as the foundation for the LDA analysis, facilitating the detection of underlying thematic patterns.
- Determination of the optimal number of topics —To determine the optimal number of topics, we employed coherence scores (C_v) and perplexity metrics. Our evaluation considered a range of topics from 2 to 15, using metrics such as those proposed by Griffiths and Steyvers (2004), Coa et al. (2009), and Arun et al. (2010). The results indicated that a model with $K = 6$ topics provides a balanced and insightful analysis. This number of topics ensures thematic granularity and interpretability without overcomplicating the results.
- Model configuration and fitting —Following the determination of the optimal number of topics, the LDA model was configured to include $K = 6$ topics. The fitting process involved executing 10,000 iterations of Gibbs sampling, with the first 200 iterations designated as the burn-in period to ensure convergence (Murzintcev, 2016).

4.2. Integration of topic and cluster analyses

To further understand the research landscape, we integrated the findings from the topic modeling with our cluster analysis. This integrated approach provides a comprehensive view of the thematic structures within our dataset, highlighting relevant overlaps and interconnections between identified topics and clusters, generating the following topics (summarized also in Table 6):

- Topic 1 (fatigue life and damage prediction) —This topic includes terms related to Cluster 1, focused on material durability, cumulative damage, and reliability. The emphasis on *fatigue* and *life prediction* indicates a relevant intersection with research aimed at understanding long-term material performance and failure prediction.

- Topic 2 (statistical distributions and regression analysis) —This topic features terms that intersect with Clusters 2 and 4, indicating a focus on statistical distributions and reliability engineering. The inclusion of *regression analysis* highlights the importance of this analysis in modeling and understanding lifetime data under various conditions.
- Topic 3 (Birnbaum-Saunders distribution and likelihood estimation) —This topic includes terms pertinent to Cluster 2, bridging the focus on statistical models and their application in reliability studies. The emphasis on *likelihood* and *maximum estimation* reflects ongoing efforts to leverage detailed statistical methods for robust reliability assessments.
- Topic 4 (Monte Carlo simulation and model estimation) —This topic features terms that align with Clusters 2 and 4, highlighting the intersection of statistical modeling and simulation. The focus on *Monte Carlo simulation* and *model estimation* further underscores the importance of understanding and predicting material behavior under various conditions.
- Topic 5 (generalized statistical distributions) —This topic includes terms strongly related to Clusters 2 and 4, emphasizing the role of generalized statistical models in reliability analysis and the need for precise *estimation* and *confidence* in these models.
- Topic 6 (umulative damage and life prediction) —This topic includes terms strongly related to Cluster 1, which emphasizes material durability and cumulative damage. The recurring themes of *cumulative damage* and *life prediction* indicate an important overlap with research aimed at predicting material failure and ensuring long-term reliability.

Table 6. Top terms in each topic identified by LDA.

Term	Topic 1	Topic 2	Topic 3	Topic 4	Topic 5	Topic 6
1	fatigue	distribution	distribution	distribution	saunders	damage
2	life	inverse	birnbaum-saunders	saunders	birnbaum	cumulative
3	damage	model	birnbaum	monte	generalized	the
4	fatigue	gaussian	likelihood	carlo	algorithm	life
5	prediction	test	maximum	estimation	distributions	and
6	prediction	analysis	data	simulation	confidence	data
7	accumulation	regression	estimation	models	maximum	model

The integration of topic and cluster analyses reveals important overlaps and interconnections within the research landscape. Topic 1 alignment with Cluster 1 underscores the critical role of understanding fatigue life and damage accumulation in ensuring material durability. Topic 2 highlights the importance of statistical distributions and regression analysis in modeling lifetime data, intersecting with Clusters 2 and 4. Topic 3 bridges statistical models and likelihood estimation, reflecting the interdisciplinary nature of these studies. Topic 4 emphasizes the role of Monte Carlo simulation and model estimation in understanding material behavior, aligning with Clusters 2 and 4. Topic 5 focus on generalized statistical distributions supports the research themes in Clusters 2 and 4. Topic 6 focus on cumulative damage and life prediction strongly supports the foundational research themes in Cluster 1. These insights are crucial for guiding future research directions, fostering collaboration, and addressing evolving challenges in statistical and reliability studies, ensuring that efforts are aligned with the most impactful and emerging areas in the field. The analysis reveals that Clusters 3 and 5, identified in the earlier section, do not align directly with any of the six main topics derived from the LDA topic modeling. This discrepancy can be attributed to the distinct nature of the analytical methods employed and the thematic structure of clusters versus topics. Clusters 3 and 5 are detailed as follows:

- Cluster 3 (estimation methods and probabilistic models) —This cluster focuses on estimation methods and probabilistic models, featuring terms like *akaike information criterion*, *bivariate*, *bootstrap bartlett correction*, *bound*, and *point estimation*. This cluster represents highly specialized and technical research areas that, despite their importance, do not emerge as main topics in the LDA analysis. The LDA tends to identify broader and pervasive themes within the corpus, whereas clusters capture specific dense sub-areas of research.
- Cluster 5 (statistical diagnostics and influence measures) —This cluster highlights statistical diagnostics and global/local influence measures, with terms such as *local influence*, *asymptotic normality*, *bias-corrected modified moment estimators*, *characteristic function*, and *consistent estimators*. Similar to Cluster 3, this cluster addresses niche specialized topics that are crucial for the field but do not appear as primary topics in the LDA results. The primary reason is that these specialized topics, while central and dense within their own network, are not widespread enough across the entire document set to form a distinct topic in the LDA model.

Furthermore, on the one hand, the topics identified by the LDA model reflect a blend of broad themes that frequently appear across various contexts within the articles. Clusters, on the other hand, tend to be more granular, representing specific sub-fields that, although critical, are less ubiquitous. Therefore, the absence of direct links between Clusters 3 and 5 as well as the six main topics is a consequence of the high degree of specialization and specificity in these clusters, which do not translate into the broader themes identified by the topic modeling. This underscores the complementary nature of clustering and topic modeling approaches: clusters can reveal concentrated areas of expertise, while topic modeling provides a higher-level overview of the main thematic currents in the research corpus.

5. Conclusions and future directions

This study has aimed to provide a comprehensive exploration of the research landscape within the domains of statistical and reliability studies, with a particular focus on the Birnbaum-Saunders distribution, Gaussian inverse distribution, cumulative damage models, and fatigue life prediction. By employing both bibliometric and thematic analyses, we have identified key trends, influential publications, and major contributors, thereby offering valuable insights into these fields.

Our analysis began reviewing 500 articles that, once filtered, reached 465 articles, selected for their relevance from the ISI Web of Science database. This dataset provided a robust foundation for understanding the evolution and current state of research. The bibliometric analysis revealed a high increase in scholarly activity over the years, peaking around 2021. This trend highlighted the growing interest and investment in statistical methodologies and their applications in reliability and lifetime data analysis.

Through our investigation, we have identified the primary outlets for research dissemination. Journals such as the *International Journal of Fatigue* and *Computational Statistics and Data Analysis* emerged as key venues for publishing high-impact research on the thematic under study. These journals have played a pivotal role in advancing knowledge and fostering discussions within the community.

Geographically, the research is widely distributed, with relevant contributions from Asia and South America, particularly Brazil and China. This global interest underscores the universal relevance of these topics and the collaborative efforts driving advancements in the field. The analysis of collaboration networks has further highlighted the strong bilateral partnerships that facilitate knowledge exchange and innovation.

The thematic analysis of keywords, supported by the Louvain method for cluster detection, provided insights into the prevalent themes and concepts. We have identified ten clusters, each representing distinct research focuses, from material durability and cumulative damage to advanced statistical methods and probabilistic models. The thematic map and cluster descriptions underscore the critical areas of investigation that are foundational to the field.

To complement the cluster analysis, we have applied latent Dirichlet allocation model to identify latent topics within the corpus. The optimal model configuration, determined through coherence scores and perplexity metrics, has resulted in six topics that provide a balanced and insightful representation of the research landscape. These topics were found to intersect importantly with the identified clusters, revealing key thematic overlaps and interconnections. However, not all clusters aligned directly with the main topics derived from the latent Dirichlet allocation model. Clusters 3 and 5, focused on estimation methods and statistical diagnostics, respectively, have represented highly specialized areas that, despite their importance, do not emerge as predominant themes in the latent Dirichlet allocation analysis. This highlighted the distinct nature of the analytical methods employed – while clusters captured specific dense sub-areas of research, latent Dirichlet allocation identified broader and pervasive themes within the corpus.

The integration of topic and cluster analyses has underscored the complementary nature of these analyses. Clusters revealed concentrated areas of expertise, while topic modeling provided a higher-level overview of the main thematic currents. This integrated approach offered a comprehensive understanding of the research landscape, guiding future research directions and fostering collaboration.

Despite the strengths of this study, limitations must be acknowledged. The choice of latent Dirichlet allocation parameters and the specific metrics used can influence the results. Additionally, our focus on the 465 most relevant articles, while ensuring a robust analysis, may exclude valuable insights from a broader dataset. Future studies could expand the corpus and explore different modeling configurations to validate and extend these findings.

Several directions for future research emerge. Expanding the scope to include a wider range of articles from various databases could provide a more comprehensive view of the research landscape. Temporal analysis of themes might offer deeper insights into the evolution of research interests and methodologies over time. Interdisciplinary approaches, fostering collaborations across different fields, may yield innovative solutions and new research directions. Exploring advanced analytical methods, such as dynamic or neural topic modeling, could provide more nuanced insights into the thematic structure of the research corpus.

In conclusion, this study provided a detailed map of the research landscape on cumulative damage models for statistical and reliability analyses, unveiling patterns and trends on the topic based on bibliometric and thematic explorations with data analytics, offering valuable insights for researchers, practitioners, and policymakers. By identifying key trends, influential works, and thematic interconnections, with the present study, we aim to guide future research efforts and foster collaborations that will advance the field, ensuring that efforts are aligned with the most impactful and emerging areas.

Author contributions Conceptualization: V.L., C.C., R.V., H.S.; data curation: V.L., C.C.; formal analysis: V.L., C.C., R.V., H.S.; investigation: V.L., C.C., R.V., H.S.; methodology: V.L., C.C., R.V., H.S.; writing —original draft: R.V., H.S.; writing —review and editing: V.L., C.C. All authors have read and agreed to the published version of the article.

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Conflicts of interest The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this article.

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