Chilean Journal of Statistics
Volume 14, Number 1 June 2023, Pages 37–47
DOI: 10.32372/chjs.14-01-03

Statistical Quality Control
Research Paper

A ladder ascending control chart with application in wind energy industry

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(Received: 20 January 2022 · Accepted in final form: 11 November 2022)

Abstract

Statistical process control is extensively applied for monitoring the production process. One of the important tools of statistical process control for detecting the unusual changes in the manufacturing process is the control chart. In the use of a control chart, samples or subgroups are selected in a manner that if unusual changes are present, the chance for differences between subgroups will be maximized, while the chance for differences within a subgroup will be minimized. However, there are different practical engineering processes where this assumption would not be applicable. In this paper, we considered a Wind Energy process where differences within a subgroup are significant and proposed a new control chart namely Ladder Ascending Control Chart under this situation. The Ladder Ascending Control Chart is based on order statistics by using order statistic distribution as the underlying distribution. Shewhart type control limits formulae for the order statistic mean have been built. It is applied to wind energy for monitoring the wind speed using the Weibull distribution as the underlying distribution of wind speed. The average run length is provided as a performance measure for the Ladder Ascending Control Chart. A real-life example is provided for its application and interpretation.

Keywords: Order statistics · Quality control · Weibull distribution · Wind speed.

Mathematics Subject Classification: Primary 62E15 · Secondary 62P30.

1. Introduction

The control chart as a statistical quality control technique is extensively used in many manufacturing processes to monitor the quality of the product. The keen interest of a quality engineer is to monitor the process variation in a manner that if the process operates with a special cause of variation then it gives an out-of-control signal. This objective is achieved with a control chart. Two major kinds of control charts are, memoryless and memory type. The memoryless chart (which uses the current information but ignores the past information) is the Shewhart provided by Shewhart (1931). It is widely used due to its simple structure and to detect the large shift in the process parameters. To detect the small and moderate shifts the memory-type control charts are effective.

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ISSN: 0718-7912 (print)/ISSN 0718-7920 (online)

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Cumulative sum (CUSUM) (Page, 1951) and exponentially weighted moving average (EWMA) (Roberts, 1959) charts are the examples of memory-type control charts. Their control structure is made in a manner that they utilize the past information along with the current to give a better performance for small and moderate shifts. To improve their sensitivity in detecting the small and moderate shifts various modified versions are proposed by many authors. For example, mixed CUSUM-EWMA due to Zaman et al. (2016), EWMA-CUSUM due to Abbas et al. (2011). Shewhart-CUSUM due to Lucas (1982) and Shewhart-EWMA due to Lucas and Saccucci (1990).

The aforementioned charts are designed on the assumptions that (i) a process being monitored will produce measurements that can be modeled with an independent and identically distributed normal distribution when only the inherent sources of variability are present in the system. (ii) the collection of sample data according to the rational subgroup concept i.e., the subgroups or samples should be selected so that if assignable causes are present, the chance for differences between subgroups will be maximized, while the chance for differences within a subgroup will be minimized (Montgomery, 2009). However, in certain applications, the process may produce measurements that can be represented with heavy-tailed distributions. In this case, the standard control charts based on normality assumption will not rapidly detect out-of-control situations, since the control limits will be stretched particularly when the nature of the products is such that one cannot take large samples to be able to use the central limit theorem. Schilling and Nelson (1976); Borror et al. (1999); Calzada and Scariano (2001); Noorossana et al. (2016); Ahmed et al. (2020); Sales et al. (2021); comment that when the monitored data do not follow a normal distribution, the usual control chart shows low performance in the monitoring.

A lot of literature is available in the context of dealing with the non-normality of process characteristics but no work is found regarding the violation of the second assumption. So in this study, we are addressing the situations in which both these assumptions are not true and proposing a monitoring technique to deal with them. In the context of the second assumption, there are situations in which differences within a subgroup are significant so forming subgroups in this situation will misguide the monitoring of the production process. Suppose a process of monitoring wind speed consists of various heights that pool their observations into a file. If we sample the wind speeds from the file by ignoring their heights then it may consist of the high and low wind speeds, moreover, it becomes very difficult to detect whether the wind speed is in-control or out-of-control. Therefore, forming subgroups in this manner fails to apply the existing monitoring technique. A logical approach to rational subgrouping here is to apply the control chart technique to the wind speed for each height. To achieve this objective, in this paper a control chart is proposed, that monitors the wind speed at all different heights simultaneously. In this situation, the rational subgroups will have ordered observations. So the control chart depicts the shape of the ladder in ascending position, therefore called the ladder ascending control chart (LACC).

After the introduction, Section 2 exposes the proposed control chart and shows its procedure in the wind energy industry using simulated data. Section 3 provides a real-life example of LACC for monitoring the wind speed and Section 4 gives a conclusion.

2. Proposed control chart with its operational procedure

The operational procedure for LACC is as follows.

Step-1: Consider the situations when quality characteristics under investigation X are measured for subgroup size n. The measuring characteristics will be in ascending order of magnitude and then written as $x_{1:n} \leq x_{2:n} \leq x_{3:n} \leq \cdots \leq x_{n:n}$, and the r-th observation in the ordered sample is denoted as $X_{r:n}$.

Assume the pivotal quantity ($y_{r:n} = x_{r:n}/\lambda.$) is the monitoring statistics. The probability density function (PDF) and cumulative distribution function (CDF) of the r-th order statistics $y_{r:n}$ are given as

$$\begin{aligned} f_{r:n}(y) &= \frac{n!}{(r-1)!(n-r)!}[F(y)]^{(r-1)}[1-F(y)]^{(n-r)}. \\ F_{r:n}(y) &= \frac{n!}{(r-1)!(n-r)!}\sum_{j=r}^{N} F(y)^j[1-F(y)]^{(n-j)}. \end{aligned} \tag{2.1}$$

Step-2: Process declares to be out-of-control for the r-th ordered observation if $y_{r:n} < \text{LCL}_r$ or $y_{r:n} > \text{UCL}_r$. On the other hand, the process declares in-control for the r-th ordered observation if $\text{LCL}_r \leq y_{r:n} \leq \text{UCL}_r$. Where LCL_r and UCL_r are the lower and upper control limits for r-th monitoring statistics. The control limits can be computed for each observation in the subgroup as

$$\begin{aligned} \text{UCL}_r &= \mu_{r:n} - L\sigma_{r:n}, \\ \text{CL}_r &= \mu_{r:n}, \\ \text{UCL}_r &= \mu_{r:n} + L\sigma_{r:n}, \end{aligned} \tag{2.2}$$

where $r = 1, \ldots, n$, L_r is the r-th control constant which will be determined by considering the in-control average run length; $\mu_{r:n}$ and $\sigma_{r:n}$ are the mean and standard deviation of ordered observations, which can be obtained using the m-th order moment as

$$\mu^m_{r:n} = E[Y^m_{r:n}] = \frac{n!}{(r-1)!(n-r)!}\int_{-\infty}^{+\infty} f_{r:n}(y)dy, \tag{2.3}$$

for $m = 2$ the $\sigma_{r:n} = \sqrt{E[Y^2_{r:n}] - (E[Y_{r:n}])^2}$.

The probability of declaring the process as out of control when the process is actually in control is denoted by p^0_r and written as $p^0_r = P[y_{r:n} < \text{UCL}_r] + P[y_{r:n} > \text{UCL}_r]$. After some evaluation, we have

$$p^0_r = 1+\frac{n!}{(r-1)!(n-r)!}\left[\sum_{j=r}^{n}[F(\text{UCL}_r)]^j[1-F(\text{UCL}_r)]^{n-j}\sum_{j=r}^{n}[F(\text{UCL}_r)]^j[1-F(\text{UCL}_r))]^{n-j}\right]$$

where $F(\cdot)$ is the CDF of the underlying distribution of the process. The efficiency of the charting plan is determined by the average run length (ARL), which indicates the average number of sample points until the process provides first out of control signal. The ARL when the process is in control is denoted by ARL_0. The ARL_0 is given as follows

$$\text{ARL}_{0r} = \frac{1}{p^0_r}. \tag{2.4}$$

Due to shift in process, the monitoring statistic fall outside the control limits and process declared out of control. The probability of declaring the process out of control for the shifted process is given as

$$
\begin{aligned}
p_r^1 =&1+\frac{n!}{(r-1)!(n-r)!}\Bigg[\sum_{j=r}^{n}(F(\mathrm{UCL}_r-\delta\sigma_{r:n}))^j(1-F(\mathrm{UCL}_r-\delta\sigma_{r:n}))^{n-j} \\
&-\sum_{j=r}^{n}(F(\mathrm{UCL}_r-\delta\sigma_{r:n}))^j(1-F(\mathrm{UCL}_r-\delta\sigma_{r:n}))^{n-j}\Bigg]
\end{aligned}
\tag{2.5}
$$

where δ denote the amount of shift. The out-of-control ARL_{1r} for the shifted process is given as follows

$$
\mathrm{ARL}_{1r}=\frac{1}{p_r^1}. \tag{2.6}
$$

The procedure of LACC will be presented in the next section.

Next, we explain the procedure of LACC in the wind energy industry based on simulated engineering process. R programming have used for simulation study and application. For the simulated data generation, we require the underlying probability distribution of the monitoring characteristic of the Wind energy industry. Wind speed is a fundamental factor for the wind energy industry, monitoring the wind turbines, modeling the air pollution, estimating the winds loads on buildings, wind power analysis, blade lifting, and modeling the wind speed variation with height for agriculture source pollution central. Therefore, the probability density function of wind speed is important for numerous wind energy applications.

Several distributions have been fitted but the Weibull distribution has received the most recent attention, see for example (Akdaq and Dinler, 2009; Morgan et al., 2011; Chang, 2011; Pishgar-Komleh et al., 2015). The Weibull distribution is by far the most applied distribution for the wind data representation, and according to the International standard of the International Electro technical Commission (IEC:614-00-12), the two-parameter Weibull is a popular approach to analyzing the wind characteristics (Gul et al., 2020).

Assume that the wind speed X has two parameters Weibull distribution with scale parameter λ and shape parameter $b>0$. The PDF and CDF of X are given, respectively as

$$
\begin{aligned}
f(x)&=\frac{b}{\lambda}\left(\frac{x}{\lambda}\right)^{b-1}\exp\left[-\left(\frac{x}{\lambda}\right)^{b}\right], \quad x>0 \\
F(x)&=1-\exp\left[-\left(\frac{x}{\lambda}\right)^{b}\right], \quad x>0.
\end{aligned}
\tag{2.7}
$$

Assume that the wind speed is measured from n different heights (lowest to highest) and let the wind speed as $x_{1:n}\leq x_{2:n}\leq x_{3:n}\leq\cdots\leq x_{n:n}$ be the order statistics corresponding to sample heights. Assume the pivotal quantity $y_{r:n}=x_{r:n}/\lambda$. Then the mean and variance of Weibull ordered random variable can be obtained by substituting the Equation (2.7) in Equations (2.1) and (2.3), we have

$$
\begin{aligned}
&E(X_{r:n})=\lambda\mu_{r:n}; \quad \text{and} \\
&E(Y_{r:n}^m)=\mu_{r:n}^m=\frac{n!}{(r-1)!(n-r)!}\Gamma\left(1+\frac{m}{b}\right)\sum_{j=0}^{r-1}(-1)^j\binom{r-1}{j}(n+j-r+1)^{-(1+\frac{m}{b})}.
\end{aligned}
\tag{2.8}
$$

Using the Equation (2.8), one may get the control limits defined in Equation (2.2). Moreover, substituting the Equation (2.7) in Equation (2.3), one can get the probability of declaring the process as out of control when the process is actually in control as

$$\begin{aligned} p_r^0 =& 1 + \frac{n!}{(r-1)!(n-r)!}\left[\sum_{j=0}^{n}[1-\exp(-[\mathrm{LCL}_r]^b)^j]\exp(-[\mathrm{LCL}_r]^b)^{n-j}\right] \\ &- \left[\sum_{j=0}^{n}[1-\exp(-[\mathrm{UCL}_r]^b)^j]\exp(-[\mathrm{UCL}_r]^b)^{n-j}\right]. \end{aligned} \tag{2.9}$$

Similary, one can get the probability of declaring the out of control for the shifted process by substituting the Equation (2.7) in (2.5) as

$$\begin{aligned} p_r^1 =& 1 + \frac{n!}{(r-1)!(n-r)!}\left[\sum_{j=0}^{n}[1-\exp(-[\mathrm{LCL}_r-\delta\sigma_{r:n}]^b)^j]\exp(-[\mathrm{LCL}_r-\delta\sigma_{r:n}]^b)^{n-j}\right] \\ &- \left[\sum_{j=0}^{n}[1-\exp(-[\mathrm{UCL}_r-\delta\sigma_{r:n}]^b)^j]\exp(-[\mathrm{UCL}_r-\delta\sigma_{r:n}]^b)^{n-j}\right]. \end{aligned}$$

Using the Equations (2.7) and (2.8), the ARL_{0r} given in Equation (2.4) and the ARL_{1r} given in Equation (2.6) can be determined. The control constant L_r will be determined for specified ARL_{0r}. The ARL_{1r} is calculated corresponding to various values of process shift δ.

Alogrithm

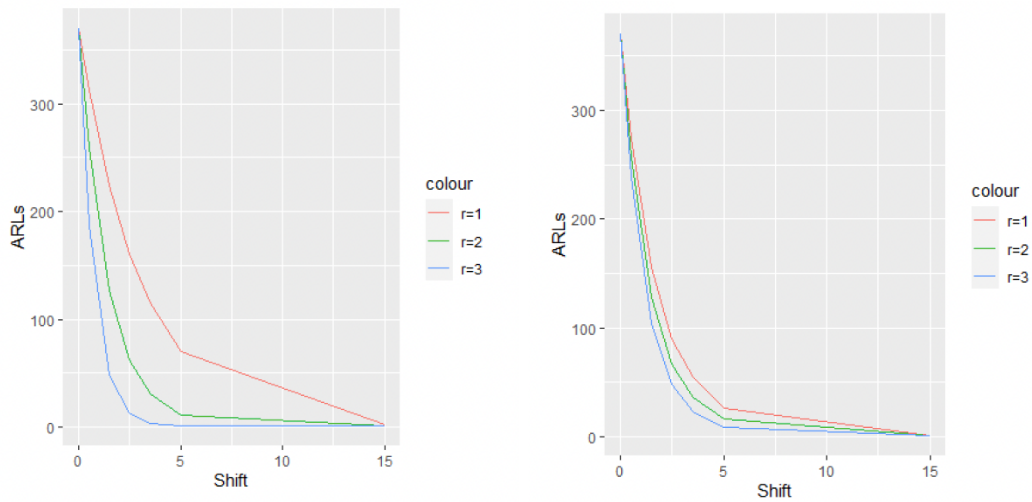
- Let $x_{1:n} \leq x_{2:n} \leq x_{3:n} \leq \cdots \leq x_{n:n}$, be the order statistics of the wind speed corresponding to sample heights for subgroup sizes $n = 2, 3, 4, 5, 7$ and 10. Assuming $b = 1, 2$ and $\lambda = 1$ because the wind speed provides extremely skewed, moderate skewed and shapes, respectively at these setting.
- Calculate the pivotal quantity $y_{r:n} = x_{r:n}/\lambda$ and compute the mean and variance using Equation (2.8).
- Set up the control limits defined in Equation (2.2) by using Equation (2.8).
- Repeat the step first three step $10,000$ times to determine the control constant L_r to get $\mathrm{ARL}_{0r} = 370$.
- Compute the ARL_{1r} values at $\delta = 0.5, 1.5, 2.5, 3.5, 5$ and 15.

Using the above algorithm, we present the control constant L_r in Table 1. The ARLs are calculated and presented in Figure 1 for the selected values of parameters for clear understanding.

Table 1. The control constant for $n = 2, 3, 4, 5, 7$ and 10 using $b = 1, 2$ and $\lambda = 1$.

		$b=1$	$b=2$
n	r	L_r	L_r
2	1	4.9136	3.3370
	2	4.5670	3.2931
3	1	4.9136	3.3371
	2	4.4307	3.2120
	3	4.4382	3.3100
4	1	4.9136	3.3365
	2	4.3841	3.1842
	3	4.2327	3.1880
	4	4.3712	3.3340
5	1	4.9136	3.3380
	2	4.3636	3.1720
	3	4.1569	3.1420
	4	4.1240	3.1835
	5	4.3301	3.3520
7	1	4.9136	3.3400
	2	4.3467	3.1650
	3	4.1025	3.1080
	4	3.9829	3.0980
	5	3.9446	3.1230
	6	4.0076	3.1954
	7	4.2824	3.3830
10	1	4.9136	3.3370
	2	4.3388	3.1570
	3	4.0780	3.0930
	4	3.9300	3.0680
	5	3.8400	3.0630
	6	3.7900	3.0710
	7	3.7760	3.0899
	8	3.8100	3.1300
	9	3.9260	3.2150
	10	4.2470	3.4235

The wind speed performance for a specific height shows the available wind potential. The ARLs values decreases with an increase in the wind tower height see Figure 1 (a) and (b). To determine the wind speed process accurately the same amount of shifts in wind speed on three different heights is applied. The shift quickly detected at highest height. For example, $b = 1$, the ARL_1 at $\delta = 1.5$ for $r = 1, 2$ and 3 respectively are $224, 124$ and 48. Which is obvious that the same amount of shift in wind speed is not equally danger at all heights. This behavior remain same for $b = 2$, see Figure 1 (b). Therefore, the wind speed characteristics varies with heights indicate the heterogeneity within rational subgroups. The proposed scheme is captured the scenario at one glance by monitoring the wind speed at various heights simultaneously. This further illustrate by applying proposed scheme to real life data set that presented in the next section.



(a) $b = 1$ (b) $b = 2$

Figure 1. The ARL values for LACC in the wind energy for $n = 3$ at various shifts using $b = 1$ and 2.

3. Real-Life Application

This study aims to propose a decision tool to define quality control criteria in manufacturing processes that have heterogeneity within rational subgroups. The considered work aims at improving the monitoring of wind speed under a given height in the wind energy industry. Making a pre-series of 6 tests at 7 different heights will allow us to note the wind speed, the datasets are available in (Chen et al., 1998) and reproduce in Table 2.

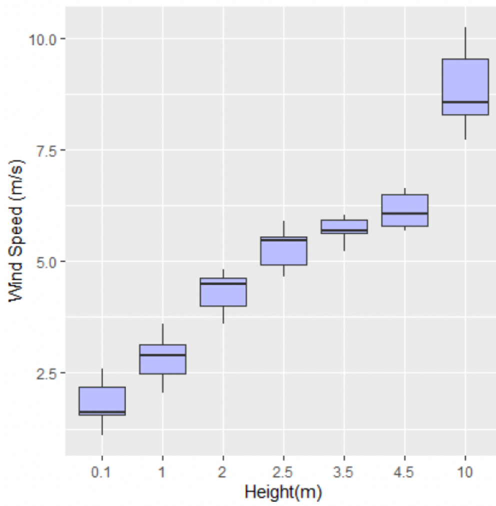
Table 2. The Wind speed at different Heights

	Heights (m)						
	0.1	1	2	2.5	3.5	4.5	10
Test	Wind speed (m/s)						
1	2.37	3.07	3.86	4.65	5.64	5.69	7.73
2	1.57	3.61	4.50	4.77	5.23	5.94	8.29
3	1.58	2.41	4.82	5.40	5.62	5.73	8.91
4	1.68	2.05	4.67	5.54	6.05	6.63	10.25
5	2.59	3.15	3.61	5.54	5.75	6.19	8.28
6	1.10	2.74	4.47	5.89	5.99	6.59	9.78

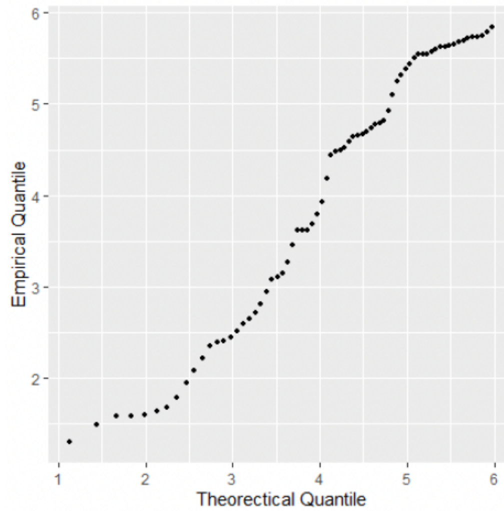
The permissible wind speed measurements are obtained by the industrial engineer specification. To achieve this goal, different stages have been carried out:

- Identify the acceptance sampling plan for this data set.
- Determine the underlying distribution of the wind speed data.
- Estimation of the parameters (method of moments).
- Expression of order statistics distributions.
- Setting up the specifications at each height.
- Computing the probability of in-control and out-of-control situations at each height.

To visualize the heterogeneity within the rational subgroups, we draw the boxplot of the wind speed data at each height. The boxplot given in Figure 2(a) shows the differences within rational subgroups are significant.



(a)



(b)

Figure 2. Boxplot of the wind speed at various heights (a); and Q-Q plot of the wind speed.

To examine the underlying distribution of the wind speed the Q-Q plot of the data is constructed and presented in Figure 2(b). The Q-Q plot indicates the wind speed approximately following the distribution in the Weibull family with shape parameter $b = 2.394$ and scale parameter $\lambda = 5.636$. Furthermore, we have tested that the sample belonging from a Weibull PDF for $b = 2.394$ and $\lambda = 5.636$ using the Kolmogorov-Smirnov test with test statistic $=$ 0.1035 and p-value=0.7592. Therefore, it is reasonable to conclude that distribution of the study variable is the Weibull distribution with $b = 2.394$ and $\lambda = 5.636$.

Moreover, the method of the moment can also be used as an alternate, the value of b and λ can be determined by the method of moments as $b \approx (0.9874/\mathrm{CV})^{1.0983}$ and $\lambda = \bar{y}/\Gamma(1+1/b)$, where CV is the coefficient of variation and $\bar{y}$ is the sample mean.

The observations of each test are obtained in ordered observations, therefore, the PDF and corresponding the CDF of the r-th order statistics for the sample of size $n = 7$ for 6 tests are given as

$$\begin{aligned} f_{r:n}(x_k) =& \frac{7!}{(r-1)!(7-r)!}\left[1-\exp\left(-\left[\frac{x_k}{5.636}\right]^{2.396}\right)\right]^{r-1}\left[-\exp\left(-\left[\frac{x_k}{5.636}\right]^{2.396}\right)\right]^{7-r} \\ & \times \frac{2}{5}\left(\left(\frac{x_k}{5}\right)^{2-1}\right)\exp\left[-\left(\frac{x_k}{5}\right)^{2}\right], \\ F_{r:n}(x_k) =& \frac{7!}{r!(7-r)!}\sum_{j=r}^{7}\left[1-\exp\left(-\left[\frac{x_k}{5.636}\right]^{2.396}\right)\right]^{j}\left[\exp\left(-\left[\frac{x_k}{5.636}\right]^{2.396}\right)\right]^{7-j} \end{aligned} \tag{3.10}$$

where $k = 1(1)6$ and $r = 1(1)7$.

We assume that the wind speed is unacceptable if $y_{(r:n)k} < \mathrm{LCL}_r$ or $y_{(r:n)k} > \mathrm{UCL}_r$, where $y_{(r:n)k} = x_{(r:n)k}/5.636$. This can occur at any given height. If it happens then it may lead to a loss for many areas, for example, agriculture crops, air pollution, estimation of wind load on buildings and wind power analysis. However, then the mean and variance of Weibull ordered random variable can be obtained by substituting the Equation (3.10) in Equation (2.1), (2.3) and (2.8).

The resulting values are used in Equation (2.2) for evaluating the control limits. Using these control limits, the probability that wind speed at a given height meets the specification is calculated using Equation (2.9) and presented in Table 3. All considered height shows that p_r^0 large enough i.e., 0.9973 that indicates the wind speed at considered height meets the specification. The value of the control constant is determined to achieve the probability 0.9973 for each height, and is given in Table 3. These intervals now clearly described the specification of the wind speed for each height and the expected quality of wind speed could be imposed.

Table 3. Result of a real-life example.

at r-th Height	LCL_r	UCL_r	L_r	p_r^0
1	-0.0412	0.9321	2.834	0.9973
2	0.1188	1.1238	3.063	0.9973
3	0.2318	1.2808	3.245	0.9973
4	0.3262	1.4337	3.391	0.9973
5	0.4001	1.6134	3.574	0.9973
6	0.4392	1.8685	3.869	0.9973
7	0.4392	2.3662	4.393	0.9973

A control chart is constructed and presented in Figure 3. In Figure 3 the control limits and level of the height appear in the same color. So that we can examine the monitoring of wind speed at a specific height. The chart shows that the wind speed at each height meets the specification. It is also observed that if we ignore the height level and consider that wind speed is taken at the same height then we construct control limits for the Shewhart chart as LCL = 2.4757 and UCL = 7.5161. The Shewhart chart is presented in Figure 4. From Figure 4 points fall outside control limits which indicates the process is out of control. So it provides misleading information if we ignore the heterogeneity within the subgroups. In this situation, LACC provides appropriate information by simultaneous monitoring of wind speed at all heights in a single chart.

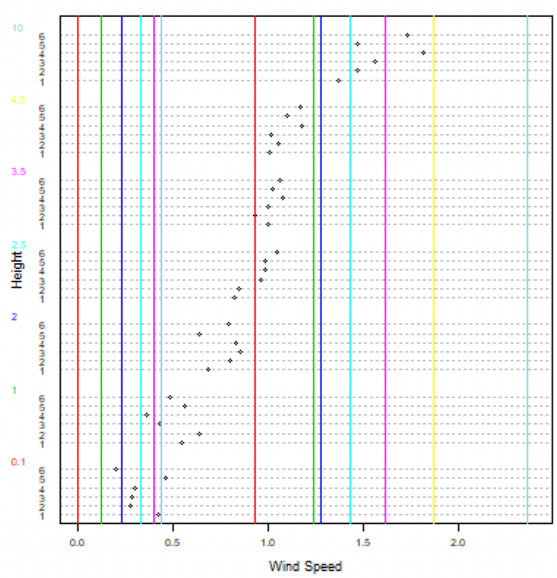


Figure 3. The LACC simultaneously monitors the wind speed at various heights.

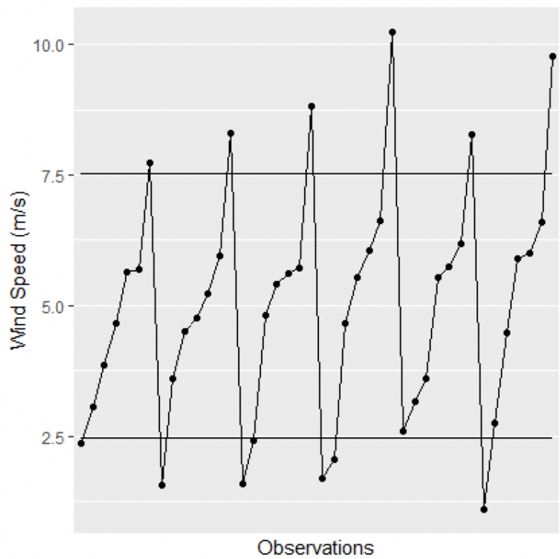


Figure 4. The Shewhart control chart ignores the heterogeneity within rational subgroups.

4. Conclusion, limitation and future research

We developed a control chart based on order statistic by using order statistic distribution as underline distribution. This type of control charts can be used to monitor the process, while the variation within a sample become meaningful and traditional chart fails to monitor. Shewhart type Control limits formulae for the order statistic mean have been built. Using Weibull distribution as underline distribution, the control limits have been obtained given different shape parameter values and scale parameter = 1, nominal ARL = 370, sample size = $2, 3, 4, 5, 7, 10$. Simulation study shows the built control charts provide simulated ARL very close the nominal ARL = 370 under the process in-control and the simulated ARL decreases when the out-of-control for shifted process. As an illustration, we consider the example of wind speed in the Wind Energy Industry in which within-sample variation of the measurement of wind speed is significant. In this situation, the LACC monitors the wind speed process more appropriately as compared to the existing Shewhart chart. The beauty of the LACC is that it monitors every observation in a random sample of size n rather than its average value. The ARLs are provided for the LACC. The limitation of the proposed chart is to use order values which become tedious to handle and may increase the cost on arranging the values for large data set. Designing the LACC with other lifetime distribution may be a fruitful area in future research.

Author contributions Conceptualization, A.A.; methodology; A.A; software, A.A.; validation, A.A; formal analysis, A.A.; investigation, A.A., M.A; data curation A.A., M.A.; writing-original draft preparation, A.A., M.A.; writing-review and editing, A.A., M.A.; visualization, A.A., M.A.; supervision, A.A., M.A. All authors have read and agreed the published version of the manuscript.

Acknowledgements We are very thankful to the referees, editor and associate editor for their valuable comments, which improved the manuscript.

Funding The authors declare that they did not receive any financial support for the research, authorship, and/or publication of this article..

Conflicts of interest The authors declare no conflict of interest.

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