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Construction of triangle meshes from images at multiple scales based on median error metric

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Abstract

The generation of models with high degree of realism has been possible through the advance of equipments and techniques for spatial data acquisition. Many applications require massive volumes of data, such as computer vision, medicine, remote sensing and virtual reality. Triangle meshes are data representations with various advantages over the use of regular grids, including adaptability to data density, ease of manipulation and visualization of complex surfaces, and organization of structures at different levels of resolution. This paper describes a method for constructing triangle meshes from images at multiple scales smoothed with Gaussian filters. A new metric for incrementally inserting data points into the mesh is proposed, which is robust in the presence of noise or outliers. Experimental results demonstrate that the proposed approach generates compact meshes while maintaining the original data surface approximation at a proper level of accuracy.

Keywords: Image smoothing · Multiscale representation · Triangle meshes.

Mathematics Subject Classification: Primary 65D18 · Secondary 68U10.

1. Introduction

The generation and distribution of spatial data sets in increasing resolutions have been possible due to the great advance of data acquisition technologies. Several applications can benefit from the use of advanced equipments for data collection. Satellites and laser scanning systems are able to capture elevation data of the earth surface at high resolution. Computed tomography, magnetic resonance, and ultrasound imaging devices acquire large image volumes of internal human body organs. Video cameras obtain sets of samples that can represent multiple views of an object.

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Efficient strategies are required for storage, manipulation and visualization of large data volumes. A common method for approximating surfaces composed of a set of points uses a polygonal mesh, which is usually represented by a set of triangles covering the domain of points (see Kirkpatrick, 1983; Garland and Heckbert, 1995; van Kaick and Pedrini, 2006). Therefore, a triangle mesh is a piecewise linear model that can be seen as a connected set of contiguous non-overlapping triangles; see Pedrini (2001) and da Silva et al. (2010).

A widely investigated problem is to create a mesh that adequately approximates a set of data points, preserving the relevant characteristics of the data and eliminating unnecessary details. The main methods for generating triangle meshes can be classified into refinement and simplification.

Mesh refinement methods (see Brown, 1997; Hoppe, 1997; Boissonnat et al., 2009; Dey and Ray, 2010) initially construct a triangulation that approximates a small number of data points. At each iteration, a new point is inserted into the mesh, typically one that presents the largest approximation error in the current triangulation according to a metric. Local adjustments are performed in the mesh and the refinement process finishes when the triangulation satisfies a given error tolerance or when a required number of points is achieved.

Mesh simplification methods (see Luebke and Erikson, 1997; Cignoni et al., 1998; Shaffer and Garland, 2001; Luebke et al., 2003; van Kaick and Pedrini, 2006) consider a mesh with all or a large number of points already belonging to the triangulation and, at each iteration, eliminate the point with the smallest approximation error. The triangulation is then locally rebuilt. As in refinement-based methods, the simplification process finishes when a required number of points is satisfied or when an error tolerance is reached.

Delaunay triangulation (see de Berg et al., 2008) is typically used to maintain the mesh, which has the property of maximizing the minimum angle of all triangles of the mesh, reducing the occurrence of long and narrow triangles that tend to cause undesirable effects, such as numerical instability and visual artifacts; see Scarlatos (1992).

This paper presents a novel metric for incrementally inserting data points into a triangle mesh, which is robust in the presence of noise or outliers. The method also generates meshes at multiple scales from a set of images obtained by the application of a Gaussian filter. Experiments show that a multiscale approach produces compact meshes according to the approximation error metric under consideration.

This article is organized as follows. The proposed method is described in Section 2. Experimental results are presented and discussed in Section 3. The conclusions of the work are given in Section 4.

2. Proposed Methodology

The method for constructing triangle meshes proposed in this work initially generates a minimal approximation consisting of two triangles over the data domain. This coarse mesh is then refined by iteratively adding new points, updating the triangulation after each point is inserted, until either a specified error tolerance is achieved or a given number of points is reached; see da Silva et al. (2010). An incremental Delaunay triangulation is used to generate the mesh.

The vertex selection criterion is crucial during the triangulation process since it determines the degree of fidelity between the original data and the its approximation. The magnitude of the error can be computed over a limited region to estimate local error or over the entire domain to measure global error. Global error measures usually produce better approximations. However, the resulting algorithms are significantly slower than those using local metrics.

One of the most common error measure used as vertex selection criterion is based on the absolute maximum difference between the actual elevation data and the surface approximation, known as maximum vertical error measure. Nevertheless, such a measure is very sensitive to the presence of noise or outliers in the data.

This work proposes a new measure for inserting points into the mesh. In each triangle, the point with median error, calculated between the original and the approximate surface. Then, the point with the highest median error for all triangles is added to the mesh. This strategy provides adequate adaptability to the data and is superior to the vertical error measure for noisy images, as will be shown in Section 3.

To insert a new point p into the mesh, its containing triangle is located, or if p lies on an existing edge, that edge is deleted and p is connected to the four vertices of the containing quadrilateral. New edges are created to connect p to the vertices of the containing polygon. All edges defining the containing polygon are checked to verify whether they satisfy the Delaunay property, that is, the circumcircle of any triangle in the triangulation must contain no other data points in its interior. If the property is satisfied, the edge remains unchanged. If it is violated, the edge is swapped with the other diagonal of its quadrilateral. In this case, two more edges become candidates for inspection. The process continues until no more candidate edges remain, resulting a Delaunay triangulation.

A priority queue is used to maintain the candidate point of each triangle, that is, the point with the highest median error. The median error is calculated by taking into the account the median absolute difference between the actual elevation data and the surface approximation for each triangle. At each refinement step, the point with the highest error within all triangles is extracted from the queue and inserted into the current mesh. Algorithm 1 shows the main steps of the mesh refinement process from a single scale image.

Algorithm 1 Mesh refinement (from a single scale image)

Input: image I, set of points P_I, maximum number of points N, and tolerance error ϵ
Output: subset P of data points and its triangulation T
$P = P_I \cup$ four corners of I
$T =$ IncrementalDelaunayTriangulation(P)
create priority queue Q with errors associated with T
while (highest median error in $Q > \epsilon$) and (number of points $< N$) do
  begin
    select point p with highest median error in Q
    $P = P \cup \{p\}$
    $T =$ IncrementalDelaunayTriangulation(P)
    update Q for the points affected by the insertion of point p
  end
return P and T

In addition to create a mesh from a single image, the proposed method constructs a triangle mesh from a multiscale image representation. In such representation, called linear scale-space (see Witkin, 1983; Lindeberg, 1994; Wang and Zhu, 2008), an image is smoothed by a sequence of Gaussian filters, generating images at different scales. A family of derived images $L(x, y; t)$ is defined by the convolution of a given image $I(x, y)$ with the Gaussian kernel

$$G(x, y; t) = \frac{1}{\sqrt{2\pi\, t}} \exp\left(-\frac{\{x^2 + y^2\}}{2t}\right),$$

such that $L(x, y; t) = G(x, y; t)I(x, y)$. The standard deviation (σ) of the Gaussian distribution is related to the scale parameter t according to $t = \sigma^2$. The scale-space representation at scale level $t = 0$ is the image I itself, that is, $L(x, y; 0) = I(x, y)$, such that the filter G becomes an impulse function. As t increases, L is the result of smoothing I with a wider filter, causing the removal of image details and noise.

A triangle mesh is initially constructed by using a predefined tolerance error over the coarser scale image. The set of points selected to construct this initial triangulation is used as a subset of points for the following scale. The use of multiple scales allows the selection of data points at coarser scales first, which are less sensitive to noise. The mesh is refined as other scales are employed. This process is repeated for the other scales until a given error tolerance is reached or a required number of points is satisfied. Algorithm 2 shows the main steps of the mesh refinement process from multiple scale representation of an image.

Algorithm 2 Mesh refinement (from multiple scale representation of an image)

1: Input: set of images I_t at scales $t = t_0, t_1, \ldots, t_s$, maximum number of points M per scale, and tolerance error ϵ
2: Output: subset P of data points and its triangulation T
3: for each image I_t, $t = t_s, \ldots, t_1, t_0$ do
4:   begin
5:     $(P_t, T) \leftarrow$ Algorithm 1 with input parameters I_t, P, M, and ϵ
6:     $P = P \cup P_t$
7:   end
8: return P and T

Figure 1 shows representations of "Lena" image at different scales and corresponding triangle meshes. Successive application of Gaussian smoothing to an image suppresses more and more details present in the finer scales of the image.

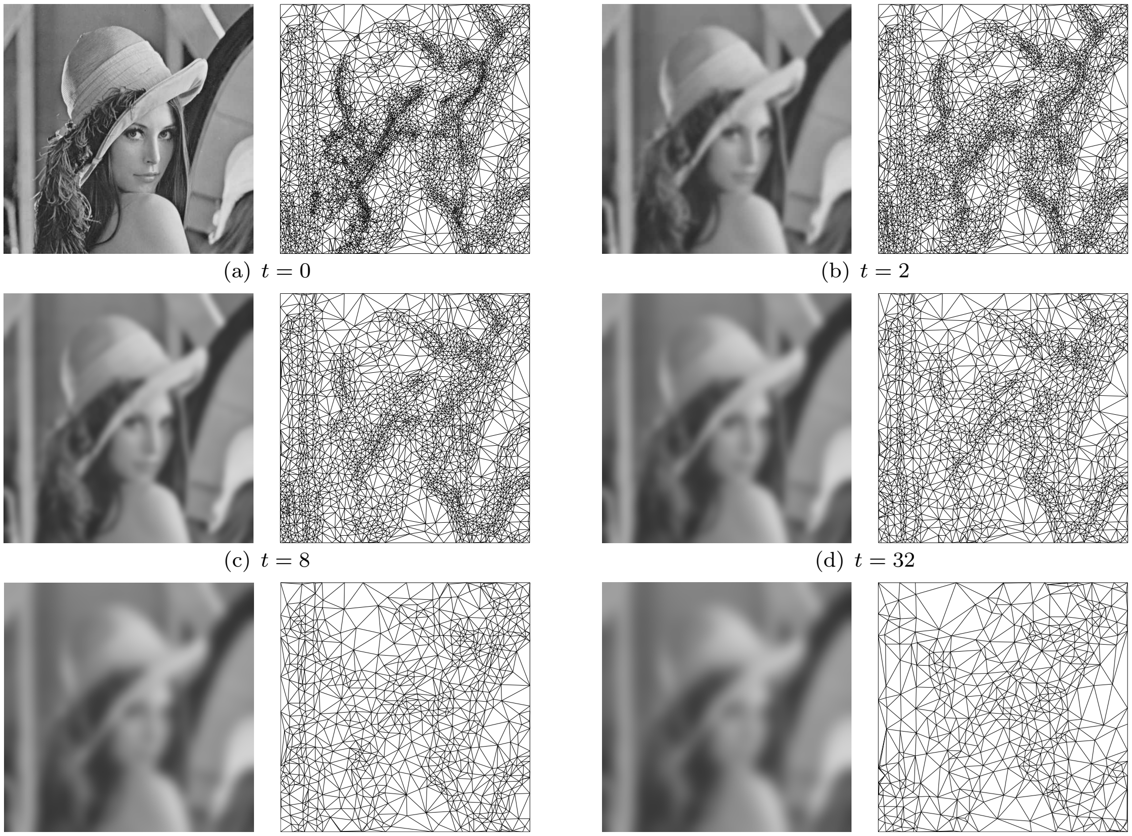
3. Results and Discussion

The median-based error measure and the method for generating triangle meshes at multiple scales were tested in a set of images with different sizes and characteristics. Figure 2 shows three standard test images, a USGS digital elevation model (see USGS, 2011), and one frame of the ETHZ pedestrian data set; see Ess et al. (2007).

Table 1 shows a comparison between the use of maximum vertical error and median error measures as metric for selecting points to the mesh constructed over "peppers" image. For each metric, the corresponding root mean square error (RMSE) is calculated with different number of points using the original image (without noise) and the reconstructed image from the mesh, which is created using the image contaminated by impulsive or Gaussian noise. It is possible to observe that the median error metric for selecting data points is less sensitive to the presence of noise in the image.

Figure 3 shows the triangulations produced with 5000 points using both error measures for "peppers" image corrupted by impulsive and Gaussian noise. The median error is able to insert more significant points into the mesh for both types of noise.

Table 2 shows the number of points inserted into the mesh and the root mean square error for each tested image using the original scale (monoscale approach) and multiple images obtained with Gaussian filter (multiscale approach). In both cases, the median error is used as metric for incrementally inserting data points into the triangulation. It can be observed that, even though the same number of points is used in both approaches, the errors for the multiscale scheme are lower compared to the monoscale scheme.



(a) $t = 0$ (b) $t = 2$

(c) $t = 8$ (d) $t = 32$

(e) $t = 128$ (f) $t = 512$

Figure 1. “Lena” image at different scales and corresponding triangle meshes.

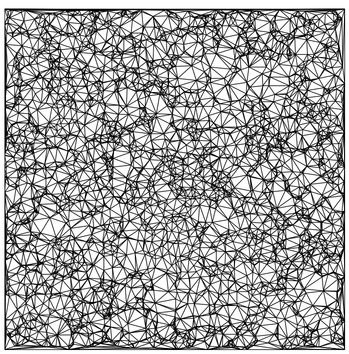
(a) Lena (512×512 pixels) (b) Baboon (533×532 pixels) (c) Peppers (512×512 pixels)

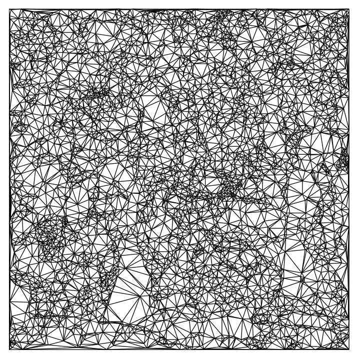
(d) Crater (336×459 pixels) (e) Pedestrians (640×480 pixels)

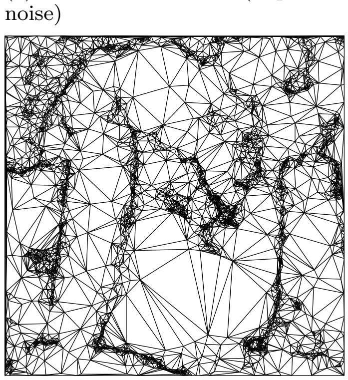
Figure 2. Set of images used in the experiments.

Table 1. Comparison between maximum vertical error and median error as vertex insertion criteria for "peppers" image. RMSE values are reported for different number of points inserted into the mesh using the original image (without noise) and the reconstructed image from the mesh.

Peppers (512×512 pixels)				
Metric	Number of points	RMSE		
		No noise	Impulsive noise	Gaussian noise
Maximum error	1,000	23.267	88.069	44.629
	2,000	15.712	81.253	39.716
	3,000	12.308	69.493	38.159
	4,000	10.817	57.733	36.676
	5,000	9.594	50.643	35.746
Median error	1,000	30.835	30.512	32.743
	2,000	24.106	25.109	27.590
	3,000	20.139	22.292	24.991
	4,000	18.411	17.175	22.598
	5,000	15.694	16.219	20.668


(a) Maximum error (impulsive noise)


(b) Maximum error (Gaussian noise)


(c) median error (impulsive noise)

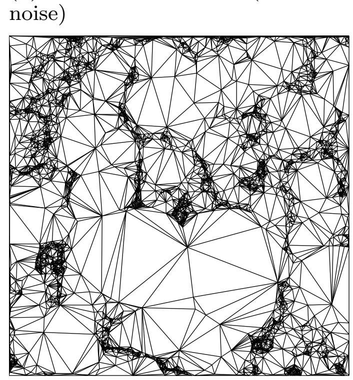

(d) Median error (Gaussian noise)

Figure 3. Triangle meshes obtained with 5000 points using maximum error and median error over peppers image corrupted by impulsive and Gaussian noise.

The maximum number of points per scale (value M in Algorithm 2) was experimentally defined in the paper. In all the conducted experiments, a maximum of 5000 points was allowed at each scale. One alternative strategy would be to define a percentage value associated with the image size, for instance, 5% of the total number of data points.

Table 2. RMSE for each tested image using both the original scale (monoscale approach) and multiple images obtained with Gaussian filter (multiscale approach).

Image	Number of points	Monoscale		Multiscale	
		Impulsive noise	Gaussian noise	Impulsive noise	Gaussian noise
Lena	5,000	17.234	22.259	15.754	19.767
	10,000	12.178	20.199	11.343	15.371
	15,000	9.669	17.995	7.565	11.454
	20,000	7.937	16.161	5.785	9.674
Baboon	5,000	31.219	33.115	29.069	32.149
	10,000	29.379	31.816	27.117	29.980
	15,000	28.691	30.679	25.495	28.153
	20,000	26.262	29.453	24.352	27.676
Peppers	5,000	16.219	20.668	13.011	17.654
	10,000	12.206	17.923	10.192	13.536
	15,000	10.543	16.014	8.646	11.982
	20,000	7.821	15.774	6.831	9.765
Crater Lake	5,000	19.698	23.872	17.342	21.315
	10,000	15.877	20.121	13.331	17.839
	15,000	11.654	17.928	10.928	15.938
	20,000	7.452	15.283	5.921	12.821
Pedestrians	5,000	30.523	31.009	28.757	30.789
	10,000	24.999	26.781	23.452	25.434
	15,000	21.745	24.135	18.989	22.874
	20,000	16.602	20.961	14.562	17.971

4. Conclusions and Future Work

A method for constructing triangle meshes from images represented at multiple scales was proposed in this paper. Data points are incrementally inserted into the mesh through a local criterion based on the median error calculated between the original and the approximate surface. Such point selection measure is robust to the presence of noise or outliers in the data. Experimental results demonstrate that meshes constructed from multiscale images smoothed with Gaussian filter approximate the original data surface with adequate level of fidelity. Although the same number of points is used in the meshes, the errors for the multiscale scheme are lower compared to the monoscale scheme.

Future work will include investigation of new schemes for generating images in different resolution levels and new criteria for selection and insertion of data points to build the mesh.

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